



Surface deformation monitoring with Kalman Filters : addressing data gaps and errors for optimal current knowledge

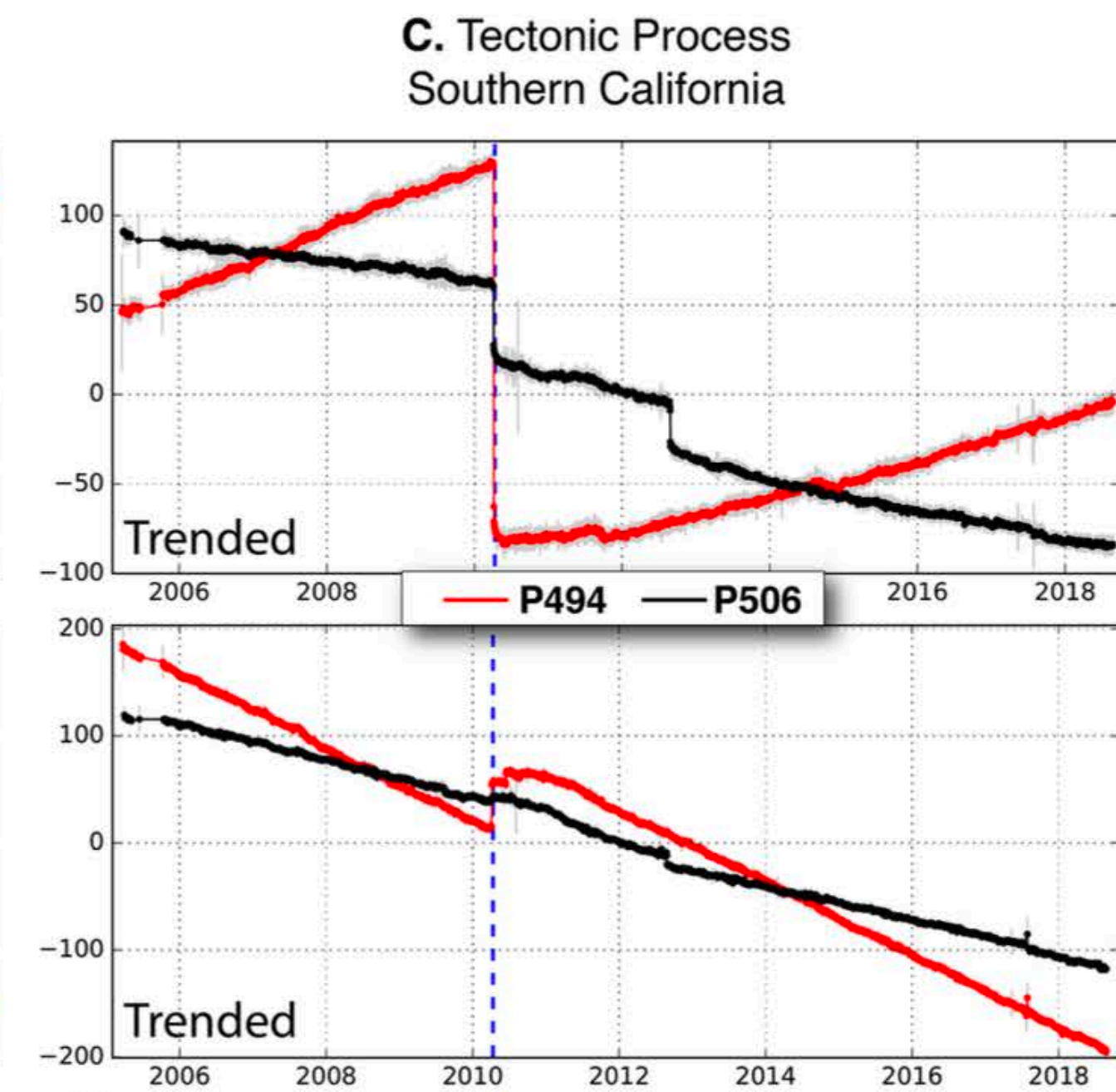
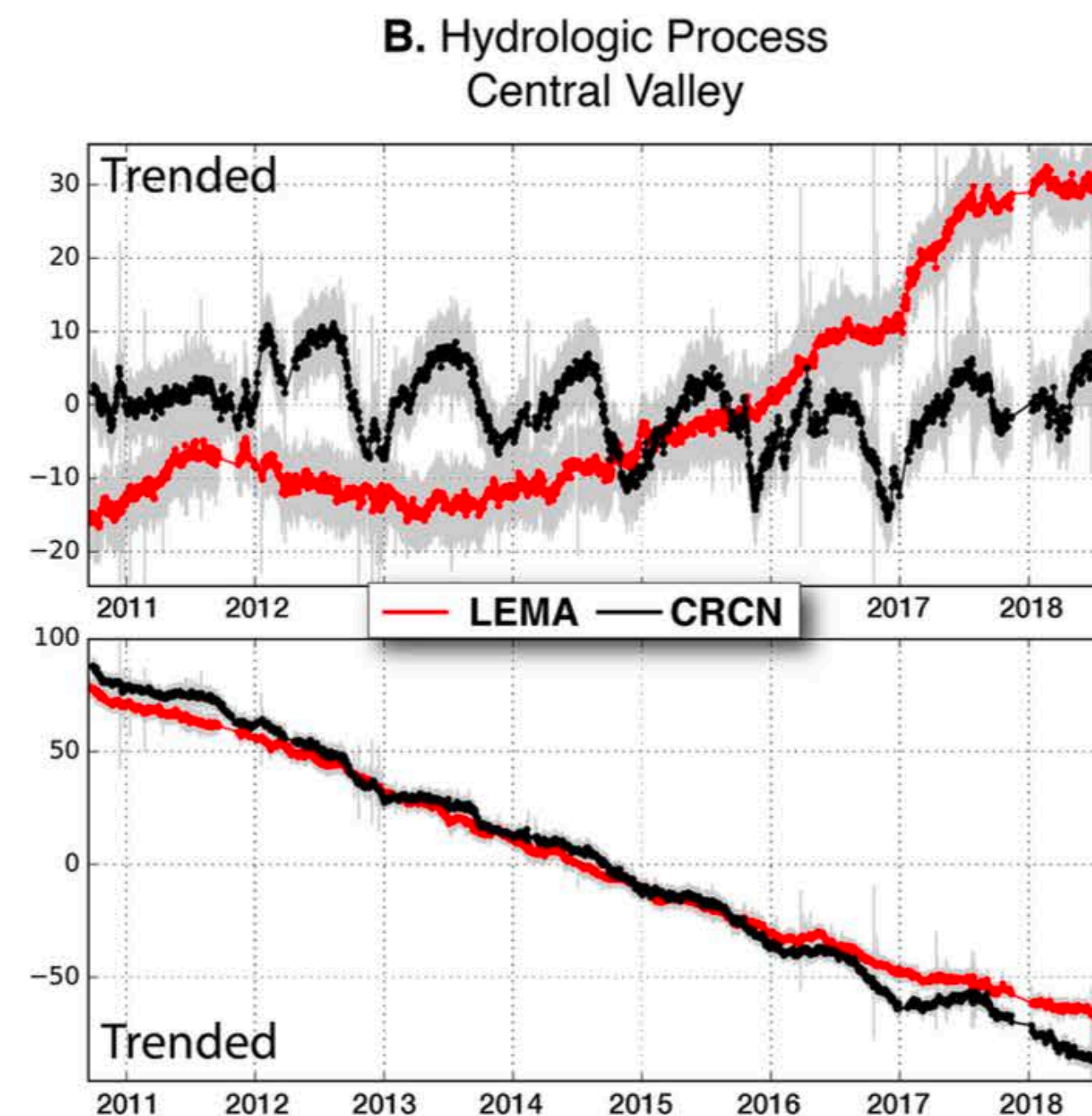
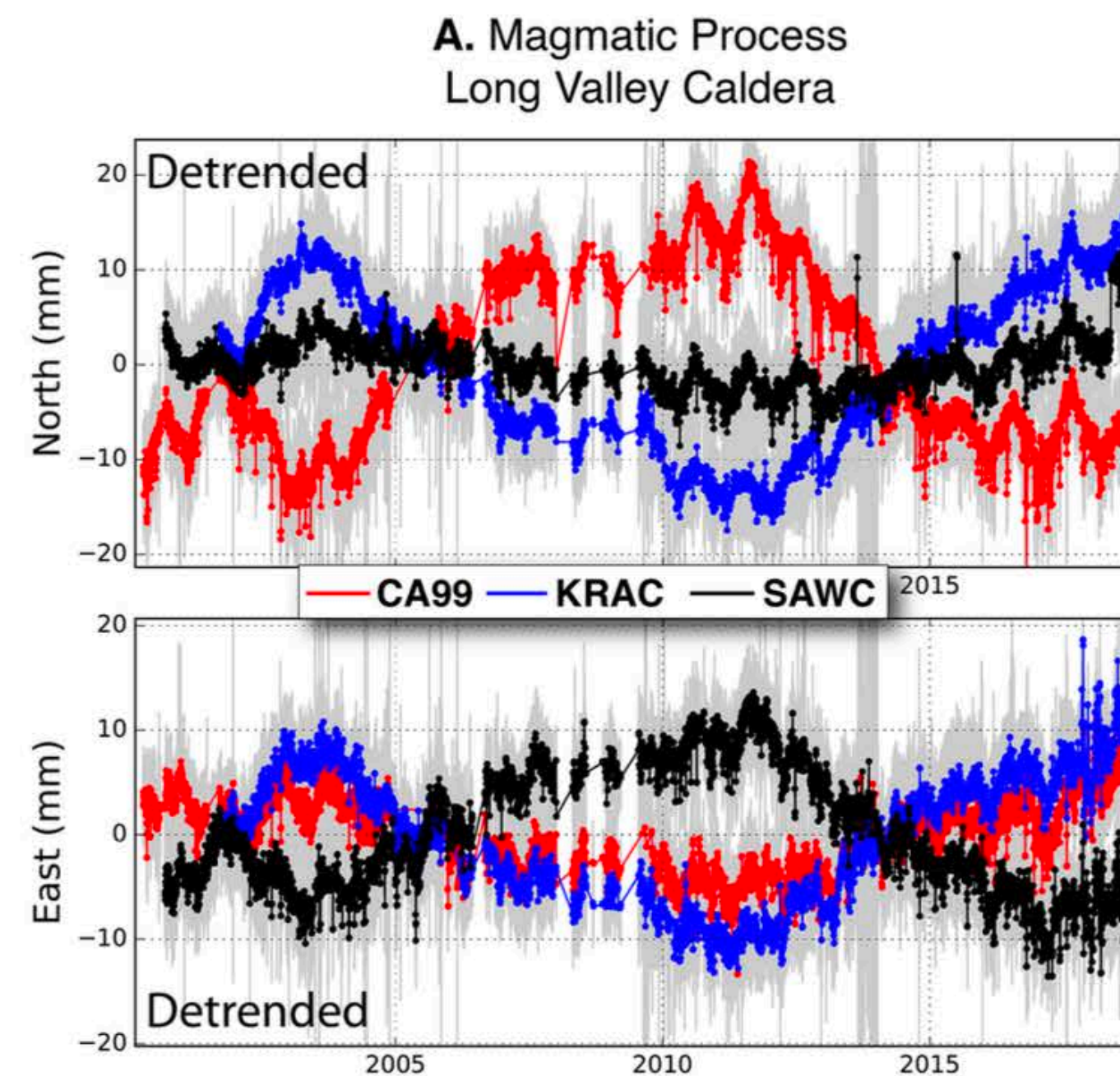
Manon Dalaison

Contributions: Beatrice Pinel-Puysségur, Romain Jolivet, Lorette Drique, Stacy Laroche, Kristel Chanard



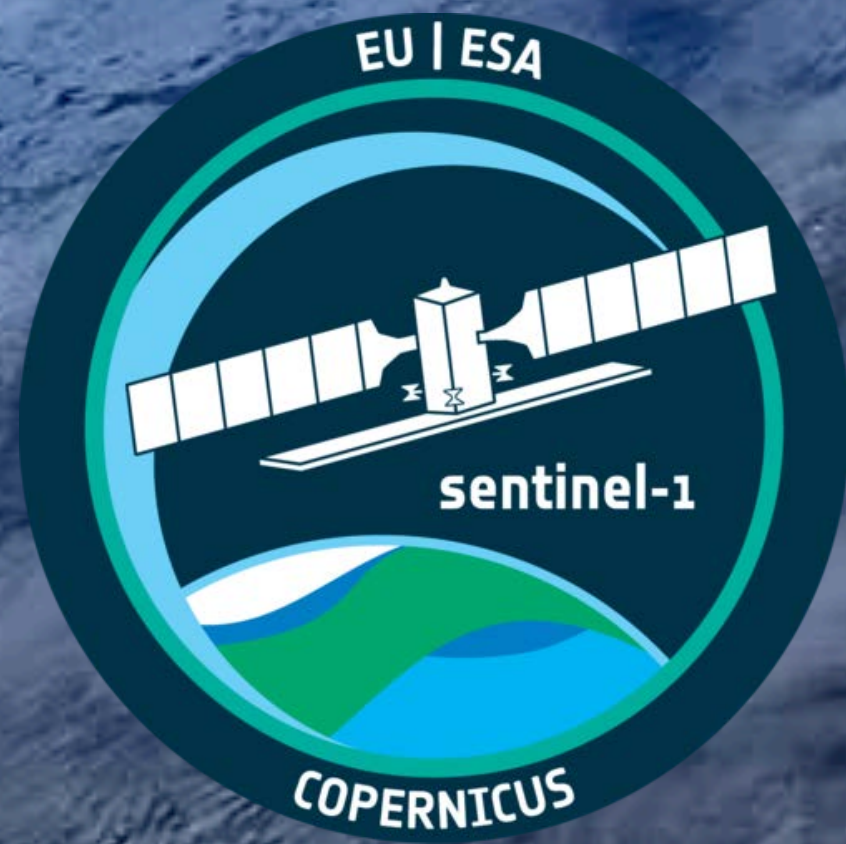
Surface deformation over time

- The ground surface moves as a result of multiples processes :
sismo-tectonics, volcanic, gravity, hydrology, anthropogenic activity
- Geodesy gives a framework and tools to measure this



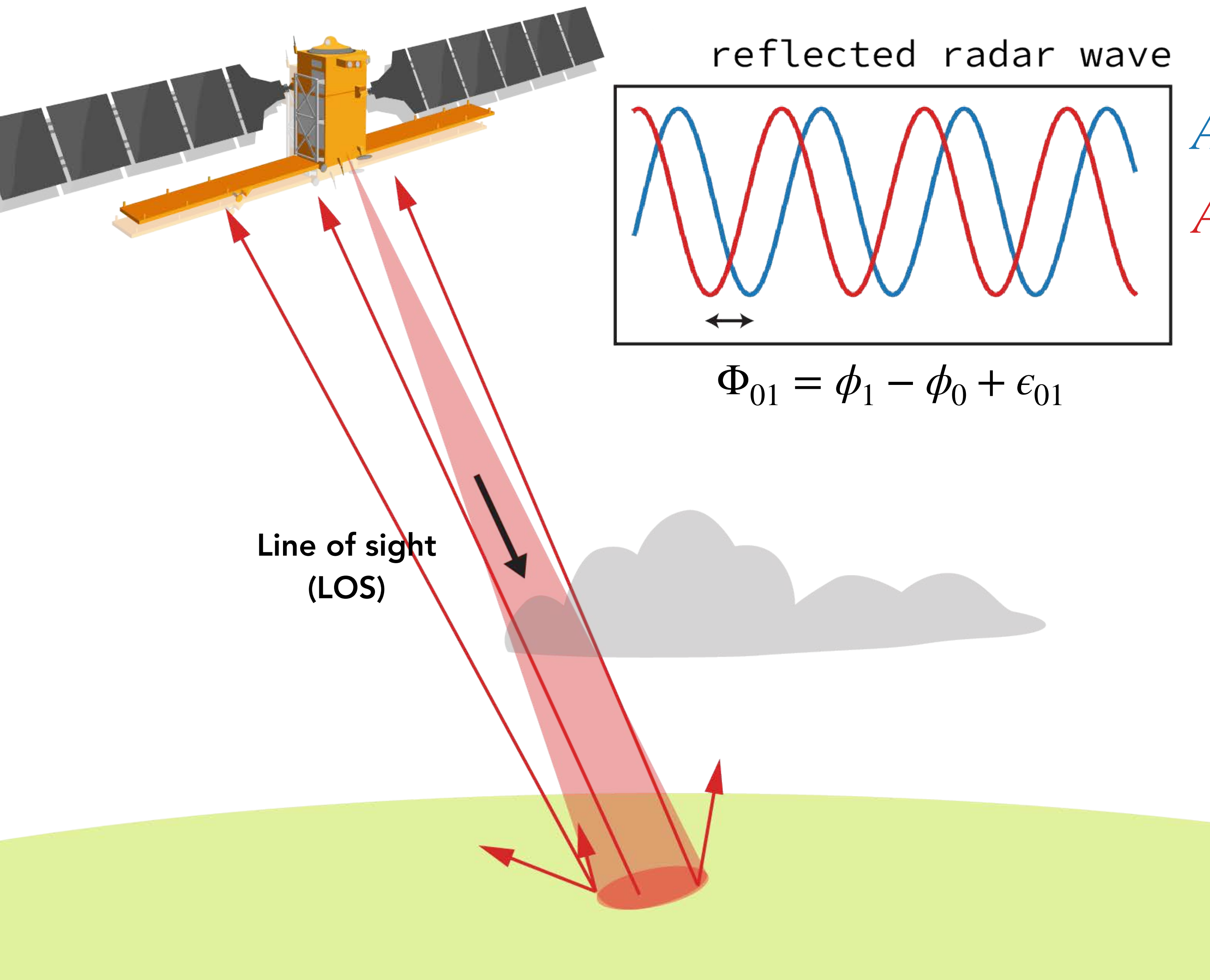


SAR

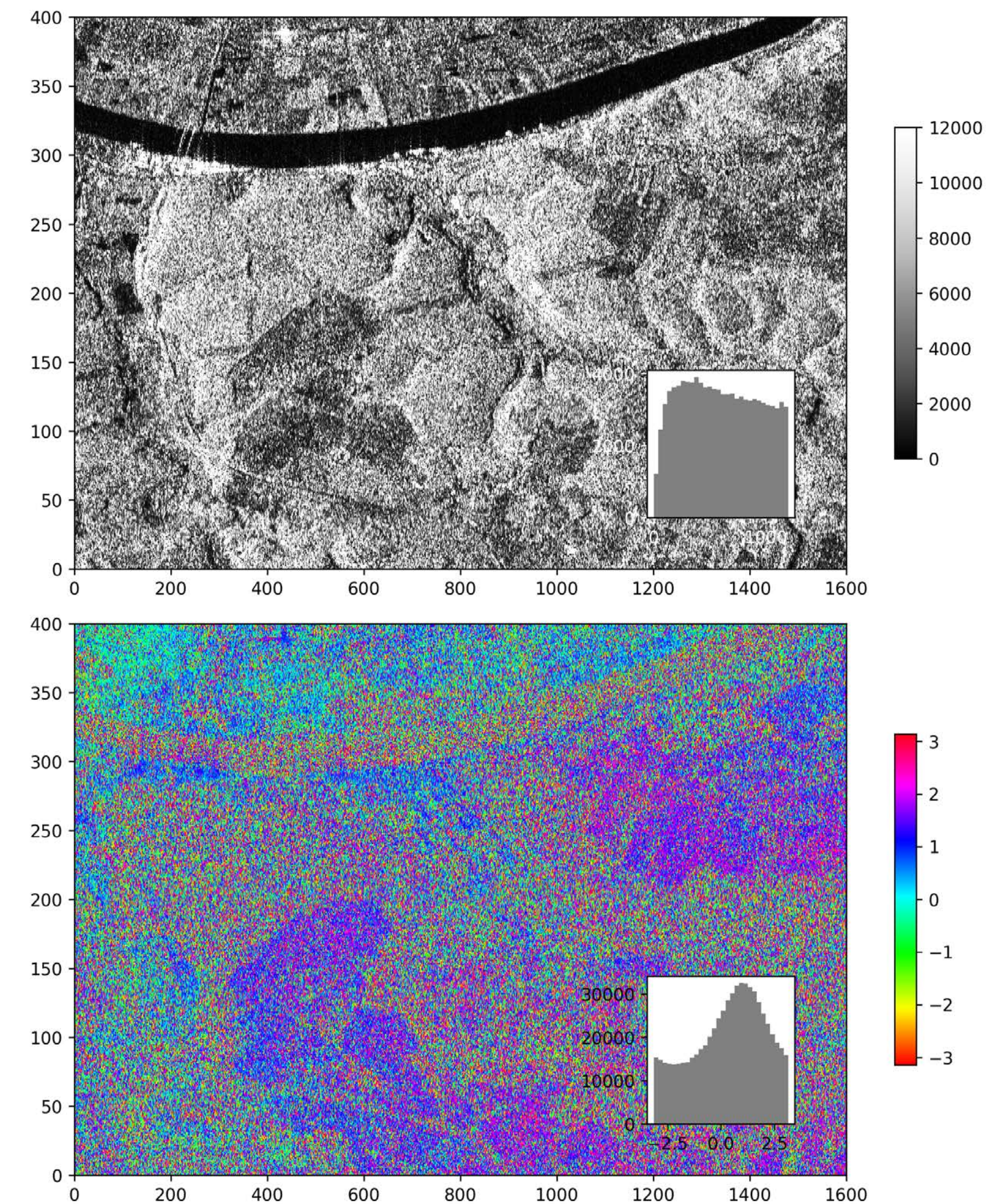


Continuous high-resolution acquisitions =
opportunity + challenge

(250 x 600 km images at 100 m resolution every 6 days)

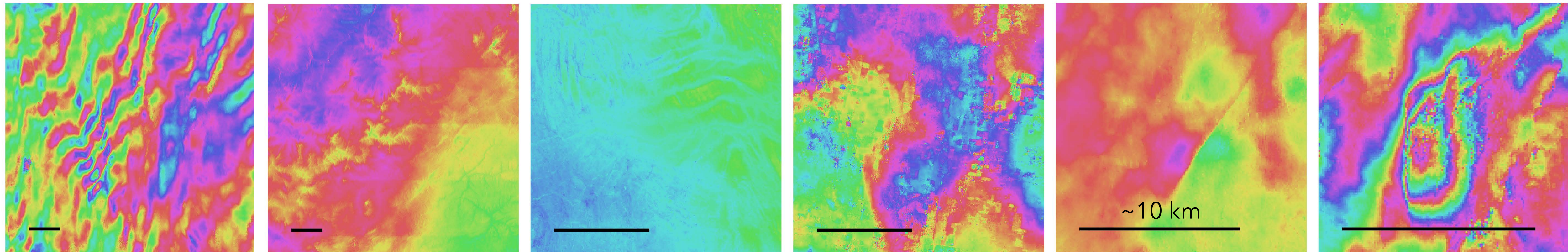
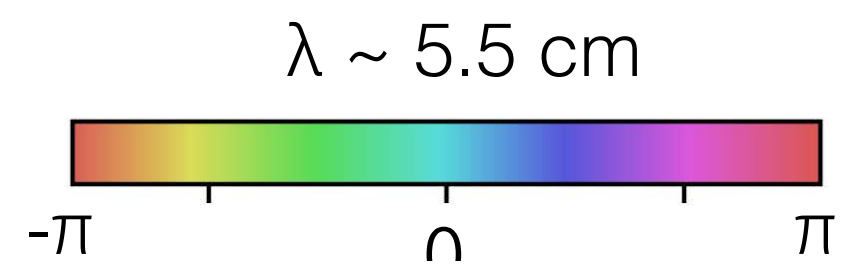
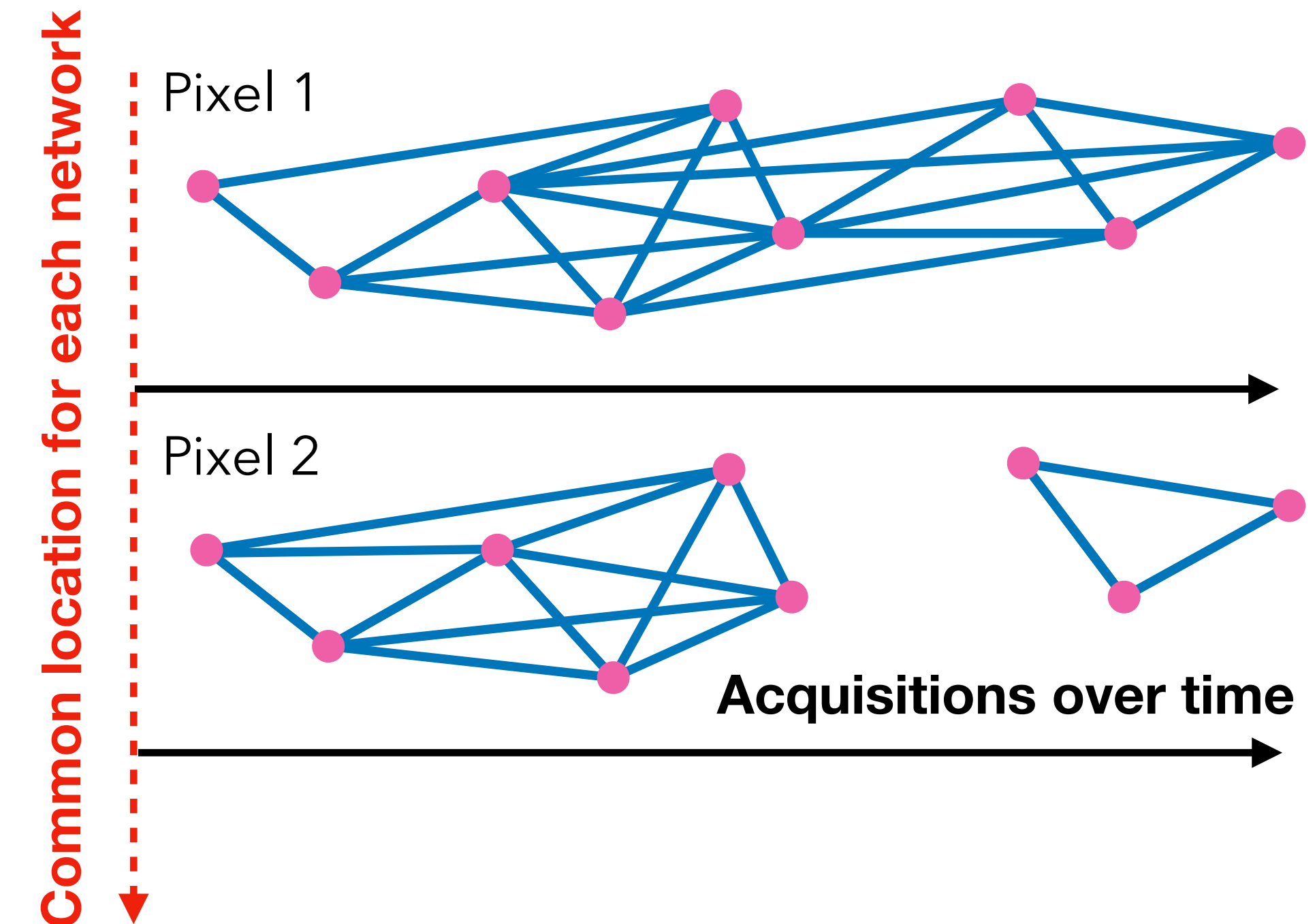


Interferometric SAR



Relative surface deformation with « errors » in LOS

- InSAR makes relative measurements as **pair-wise differences** in time
- Phase change = **deformation** + relative position + atmosphere + soil moisture + ...
- Indirectly linked to geology, climate, weather, agriculture.



Big data challenge

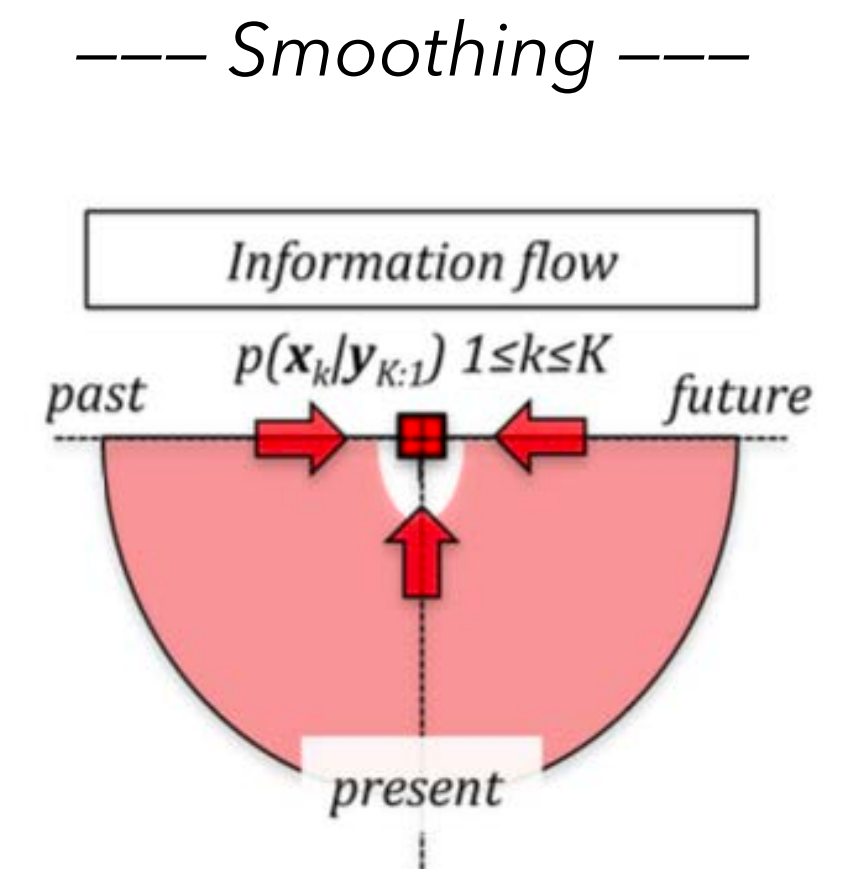
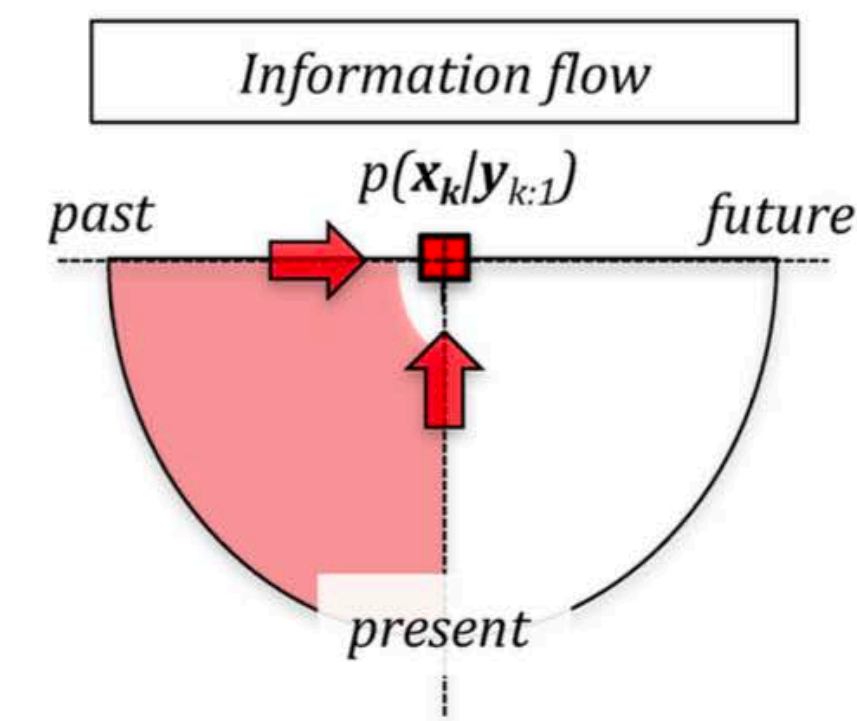
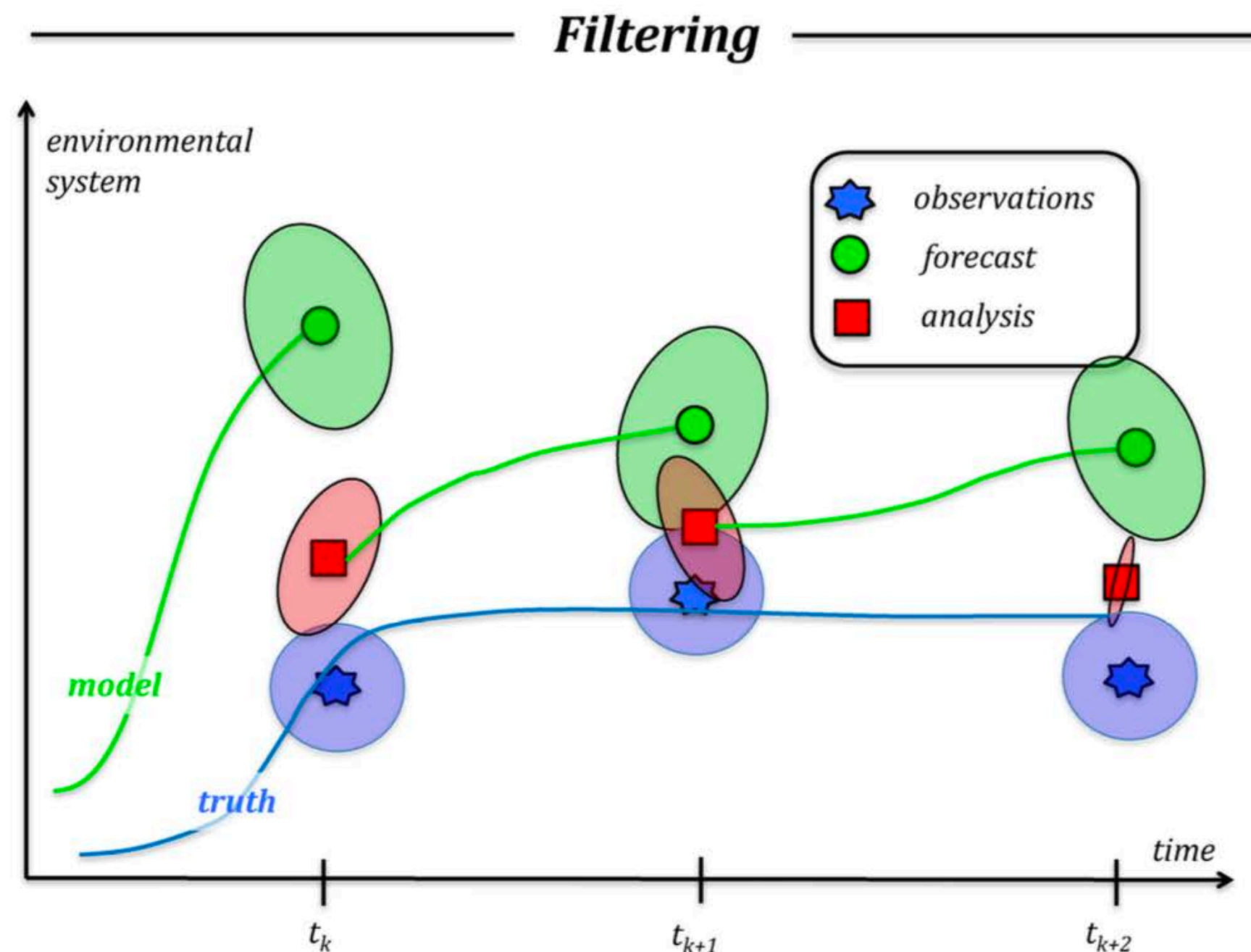
- InSAR requires a computing infrastructure
- Data volume optimization: need to delete old files
- Data quality tracking:
 - Relative measurements in time and space, error build-up and multi-origin signal
 - White and colored noise with correlations

Shared services / S-CAPAD

> How can we track ground deformation efficiently and accurately ?

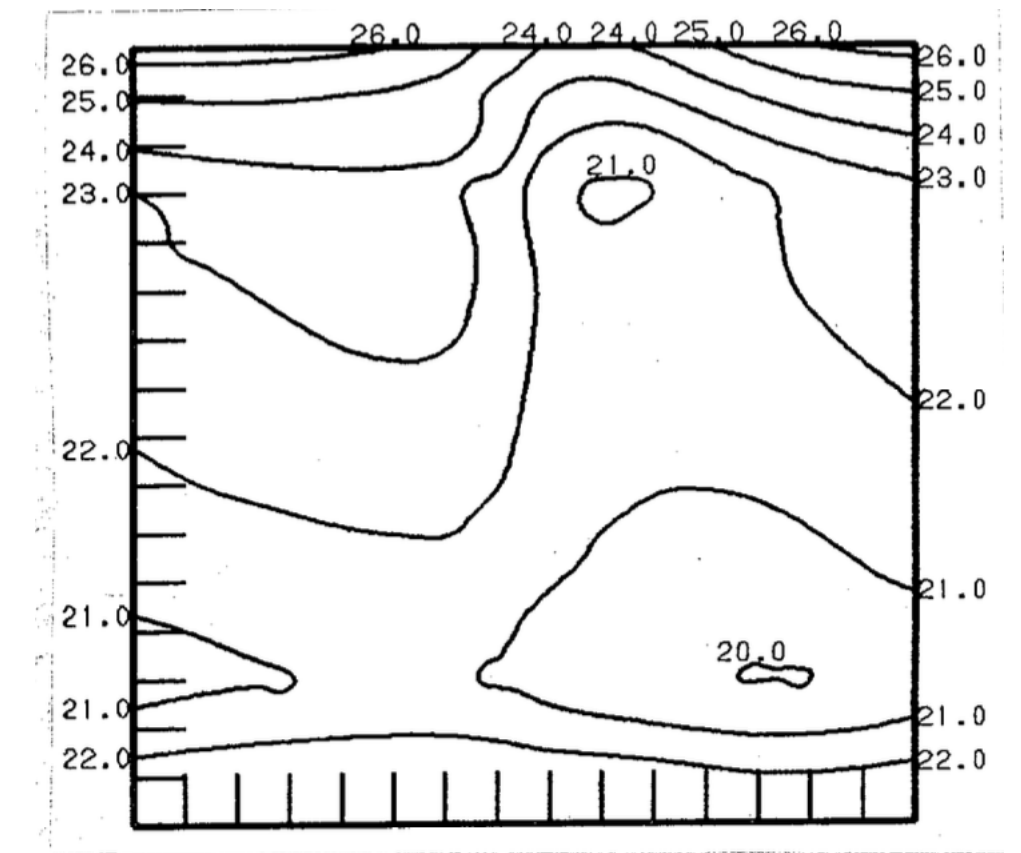
Data Assimilation in (very) short

- State-estimation theory
- Combine two complementary, but incomplete and inaccurate sources of information : the observations and the model
- Discrete/iterative estimations from past, present and eventually future informations

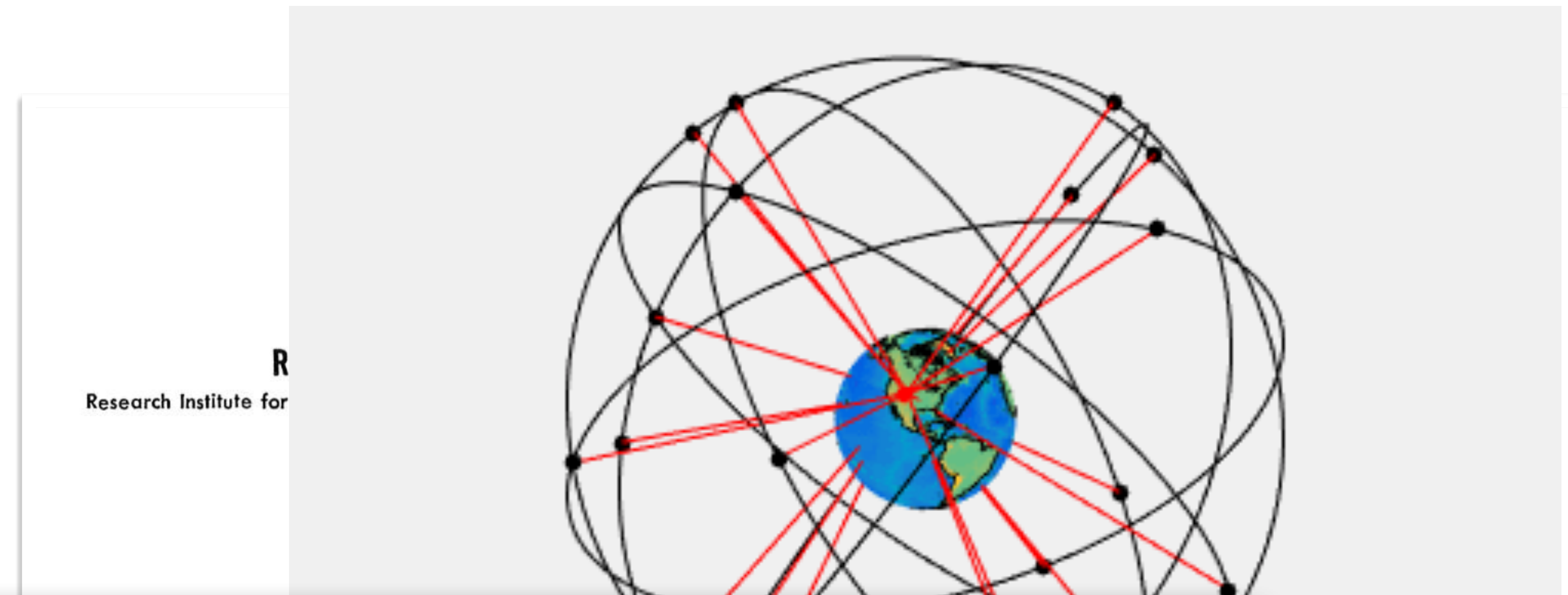


Kalman filter

- Computationally efficient, sequential optimal estimates
- For Gaussian & linear problems
- Designed to provide initial conditions for numerical weather predictions.
- Many adaptation: EKF, EnKF...
- Used to build GNSS time-series (ex. GLOBK "Global Kalman filter »).
State includes: station coordinates, Earth orientation parameters, orbital parameters, atmospheric parameters



Parrish & Cohn 1985



> Optimization to produce the best deformation « observations » knowing the data acquisition structure.

In this presentation

- A. Introduction to the design and parameterization of the Kalman Filter
Time series analysis for InSAR
- B. Discussion on the complex nature of errors in InSAR phase change
- C. Some examples of deformation monitoring
- D. Limits and conclusions

Monitoring deformation with InSAR

Governing equations

Data (Interferograms)

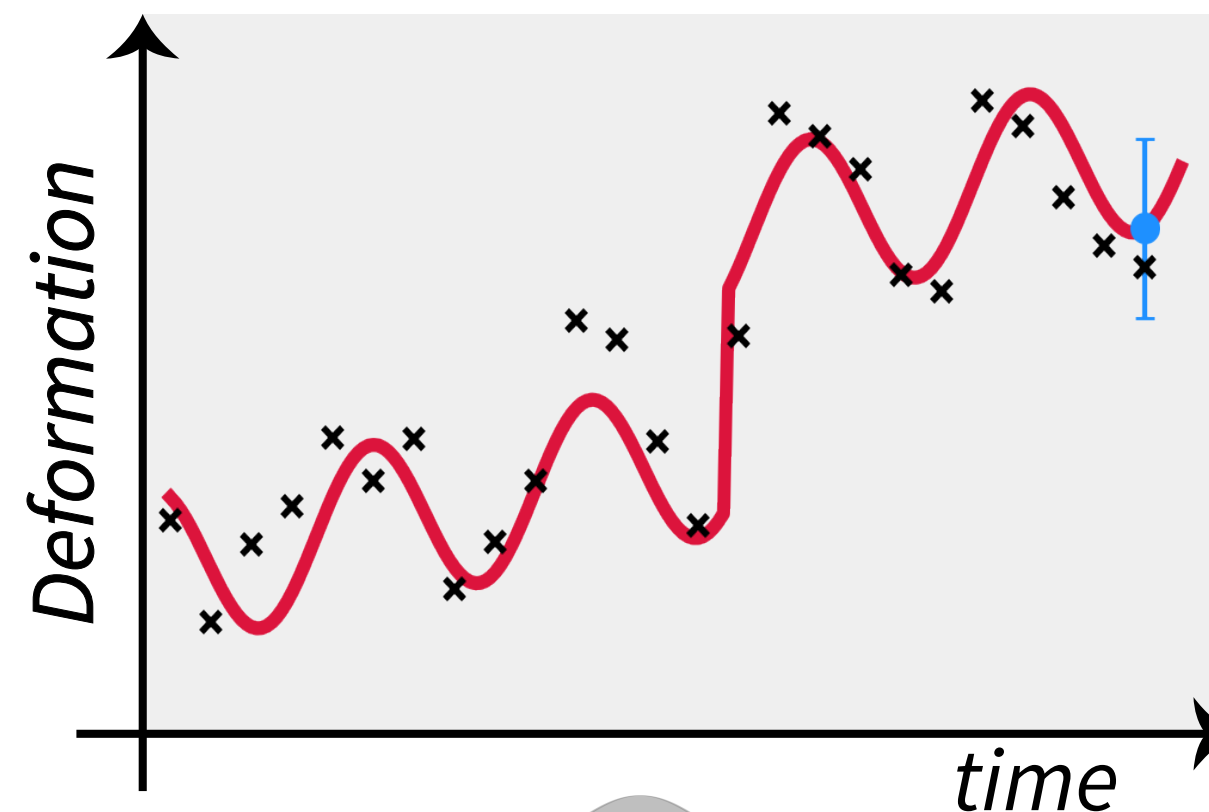
$$\Phi_{ik} = \phi(t_k) - \phi(t_i) + \epsilon$$

error

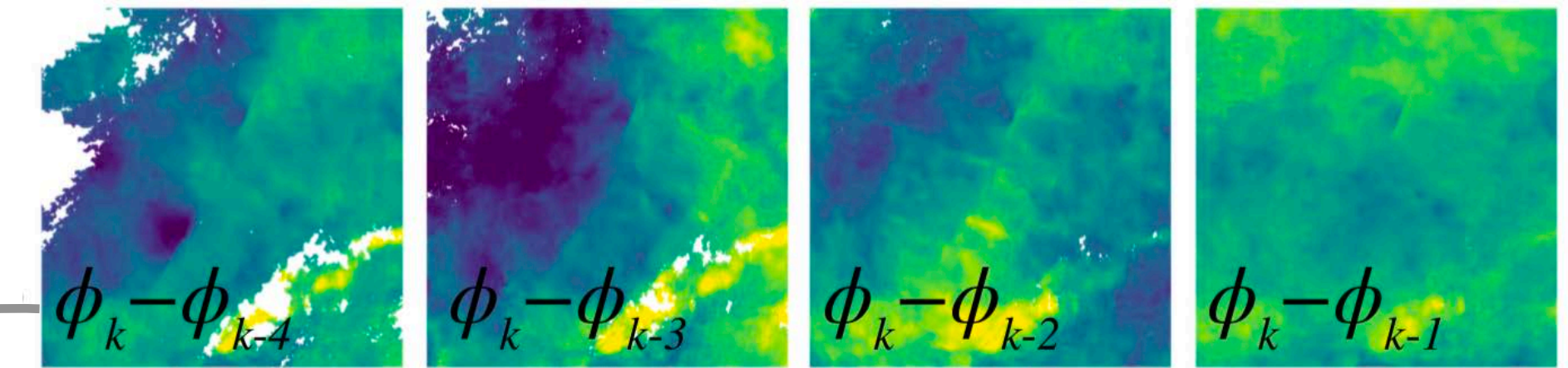
Mismodeling

$$\phi(t_k) = \sum_{n=1}^N a_n f_n(t_k) + \gamma$$

Trajectory model
per pixel

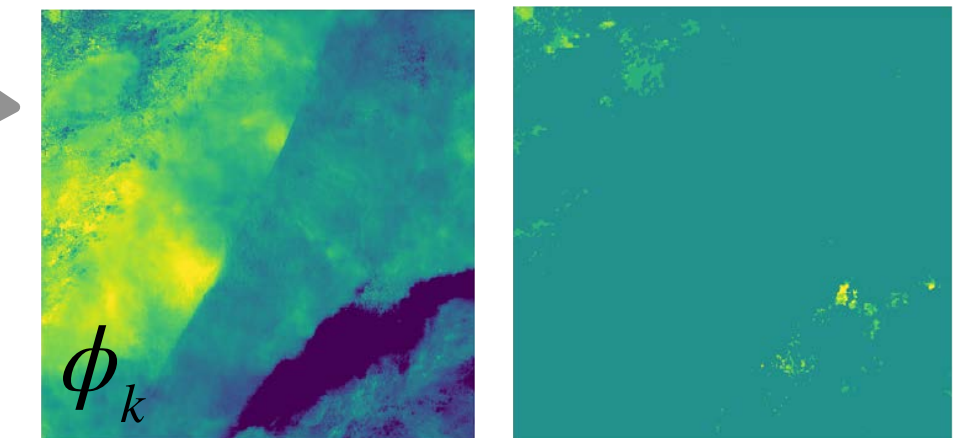


Forecast
 $\phi^f(t_k)$



New data : unwrapped interferograms

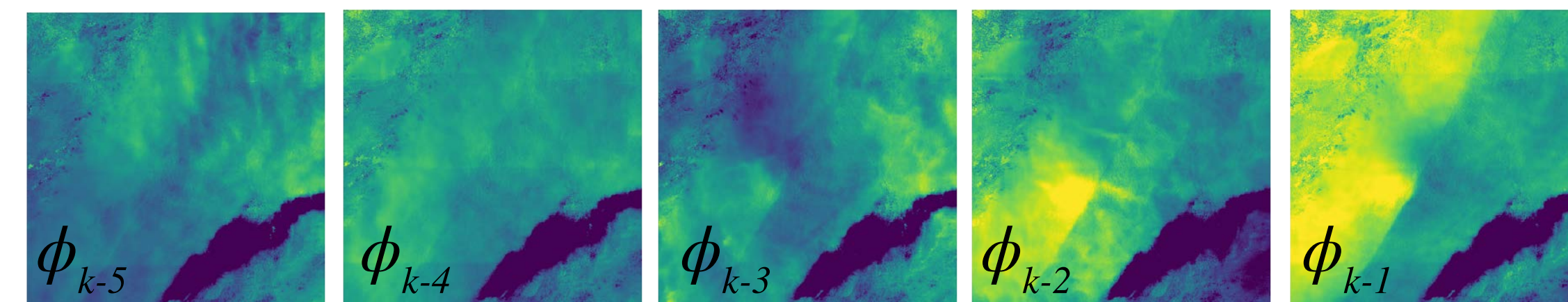
$\phi(t_k)$
Analysis (new
phase change
estimate)



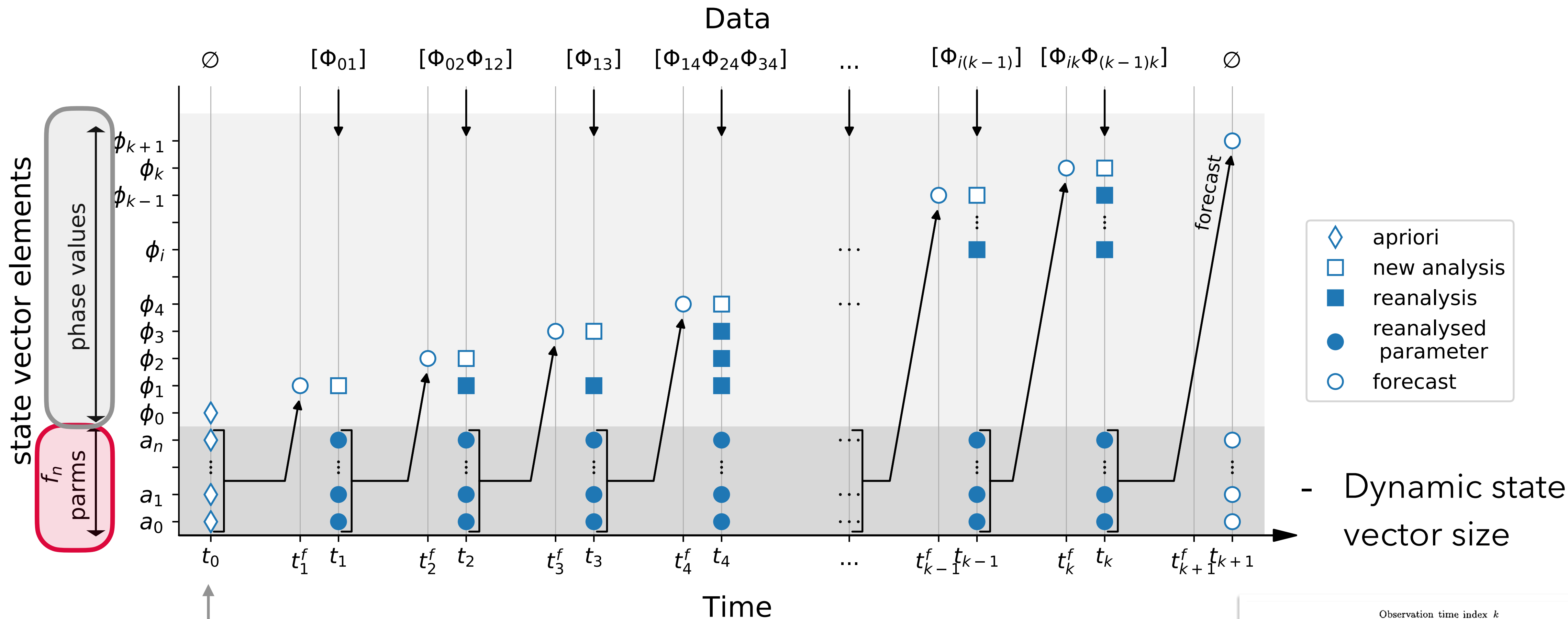
ϕ_k standard
deviation

Re-analysis
(update)
 $a_n, \phi(t_i)$

Iterative process
 $t_k \rightarrow t_{k+1}$



Time series of phase change



P_0
 m_0
 A priori

$$m_k^f = A_k m_{k-1}$$

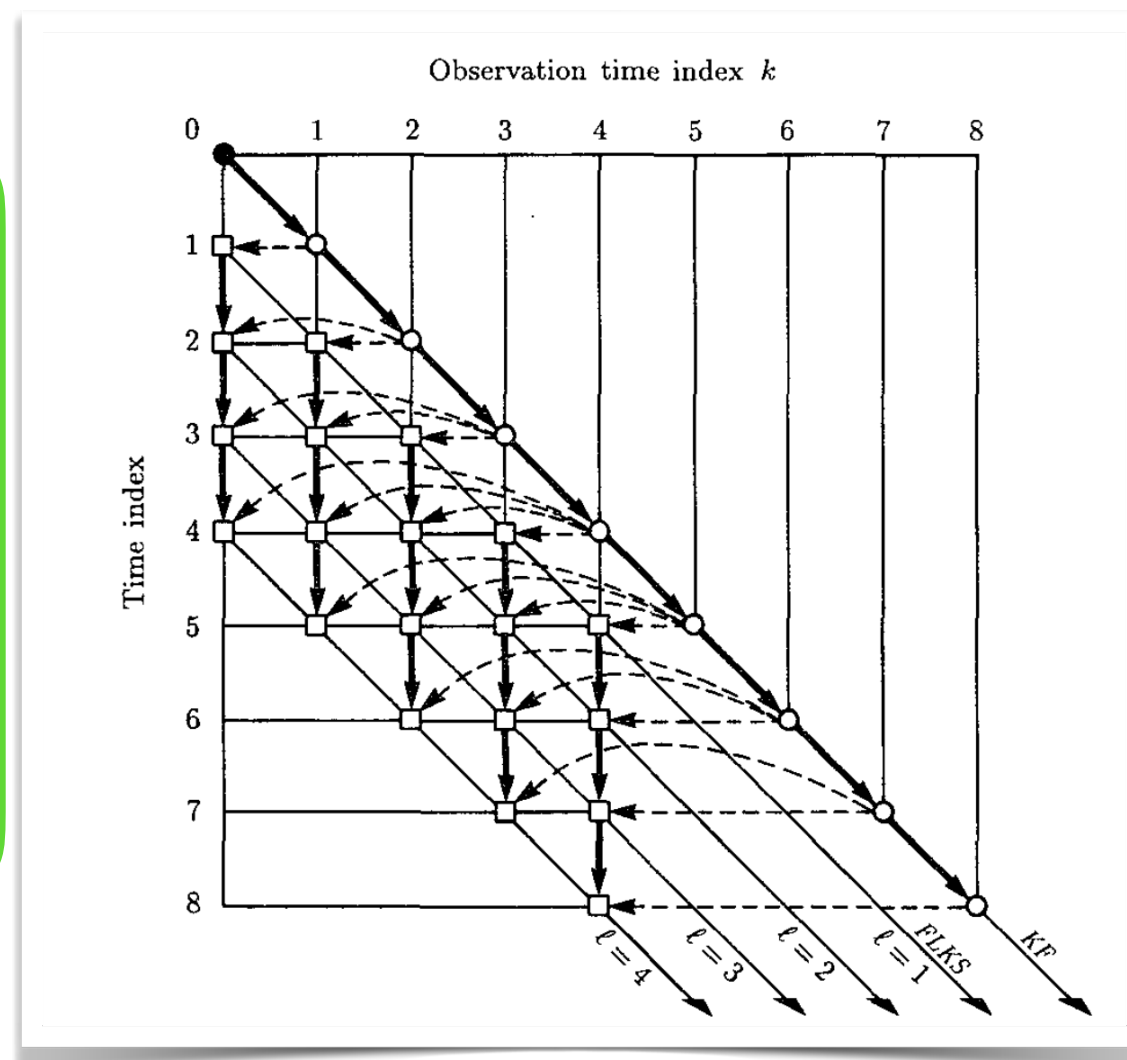
$$P_k^f = A_k P_{k-1} A_k^T + Q_k$$

- Forecast

Mismodeling

~Smoother with the use of future data in reanalysis

Cohn et al. (1994)



A forecast based on a linear trajectory model

$$\phi_k = a_0 + a_1 t_k + a_2 \sin\left(t_k \frac{2\pi}{T_{year}}\right) + a_3 \cos\left(t_k \frac{2\pi}{T_{year}}\right) + a_4 S_{sse}(t_k) + a_5 H_{eq}(t_k) + \gamma_k,$$

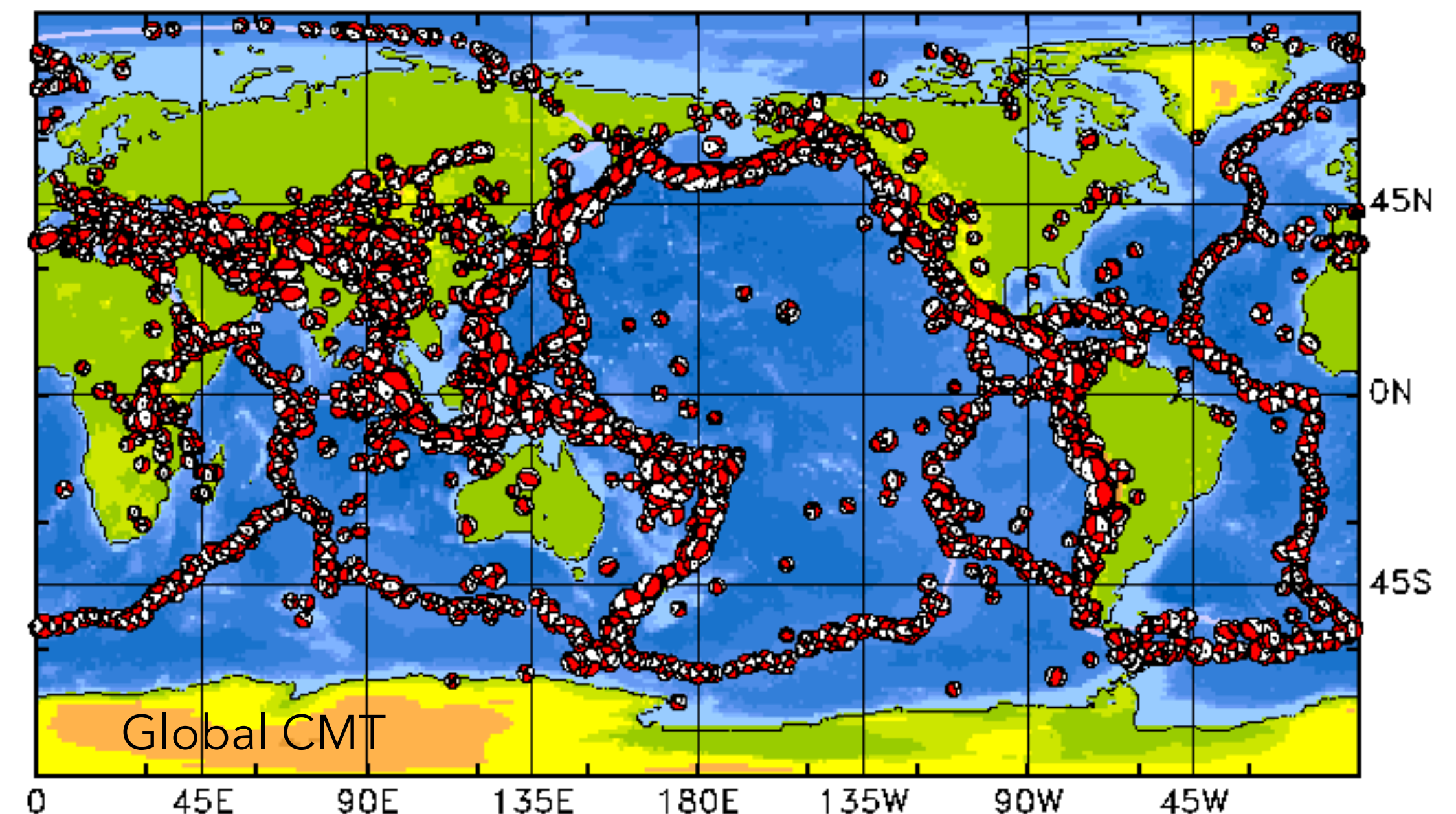
Error and noise in the
reference image

Deformation
rate

Seasonal oscillation

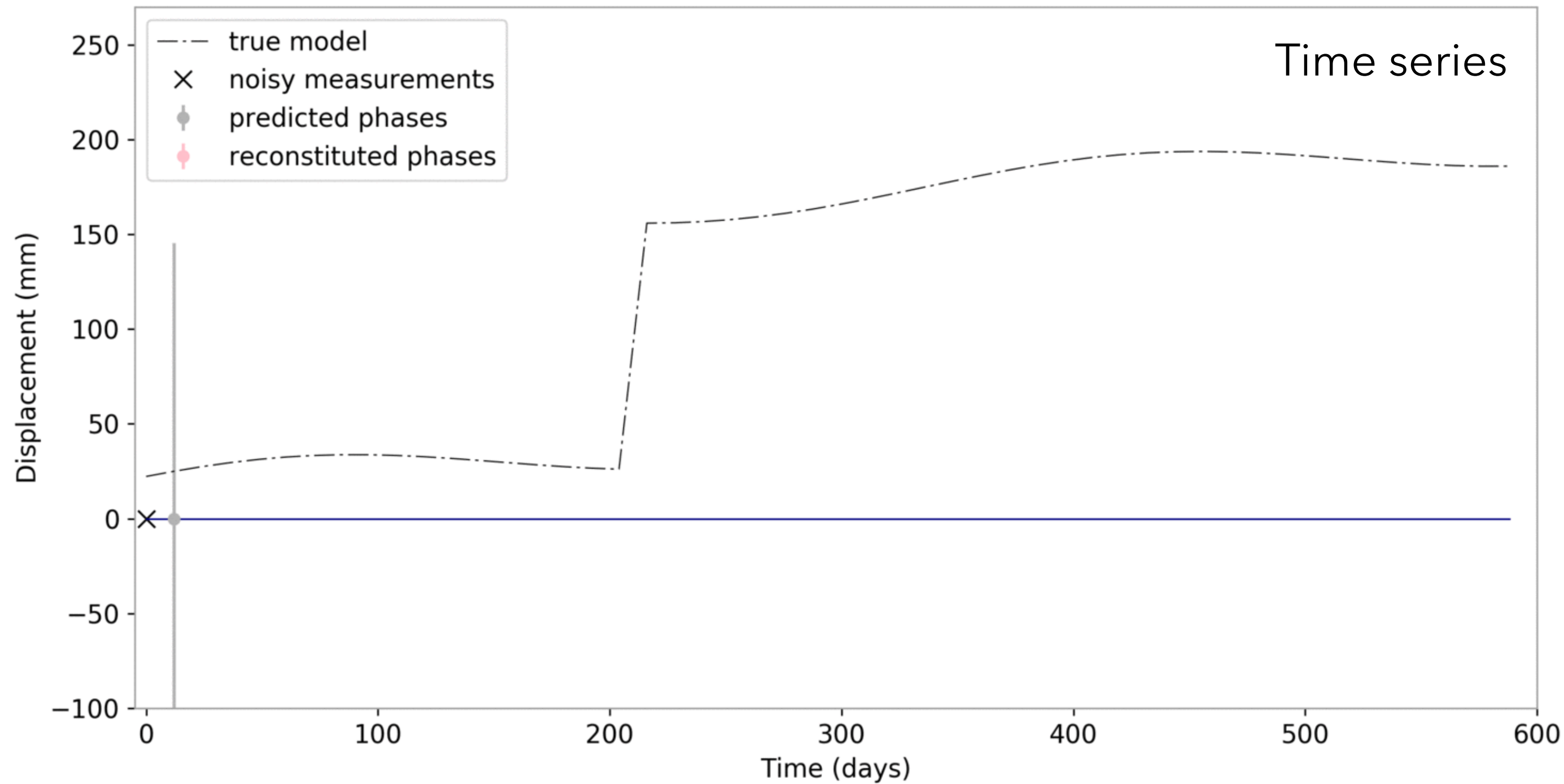
Earthquake
"coseismic" motion

- Seismic catalogs with EQ location and timing



- Potential Improvements
 - Functions depending on satellite acquisition conditions (spatial baselines, temperature...)
 - Explicit time-varying parameters

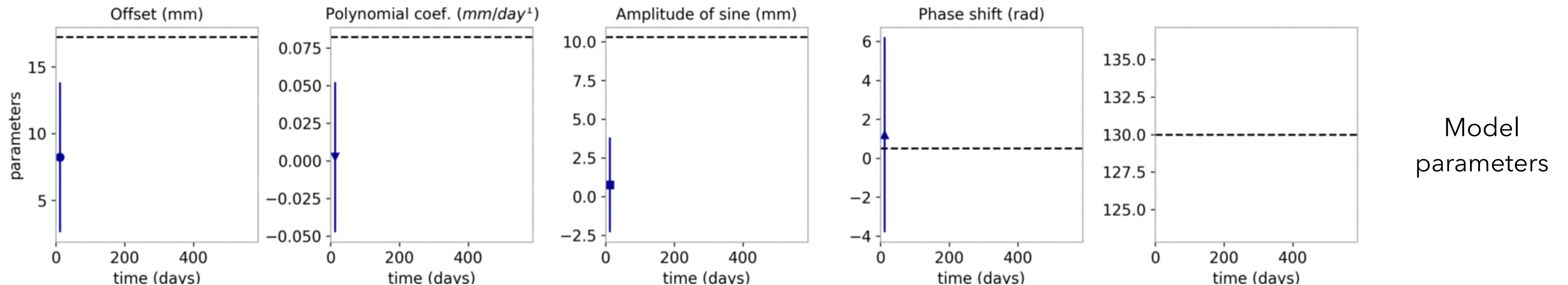
KFTS for InSAR



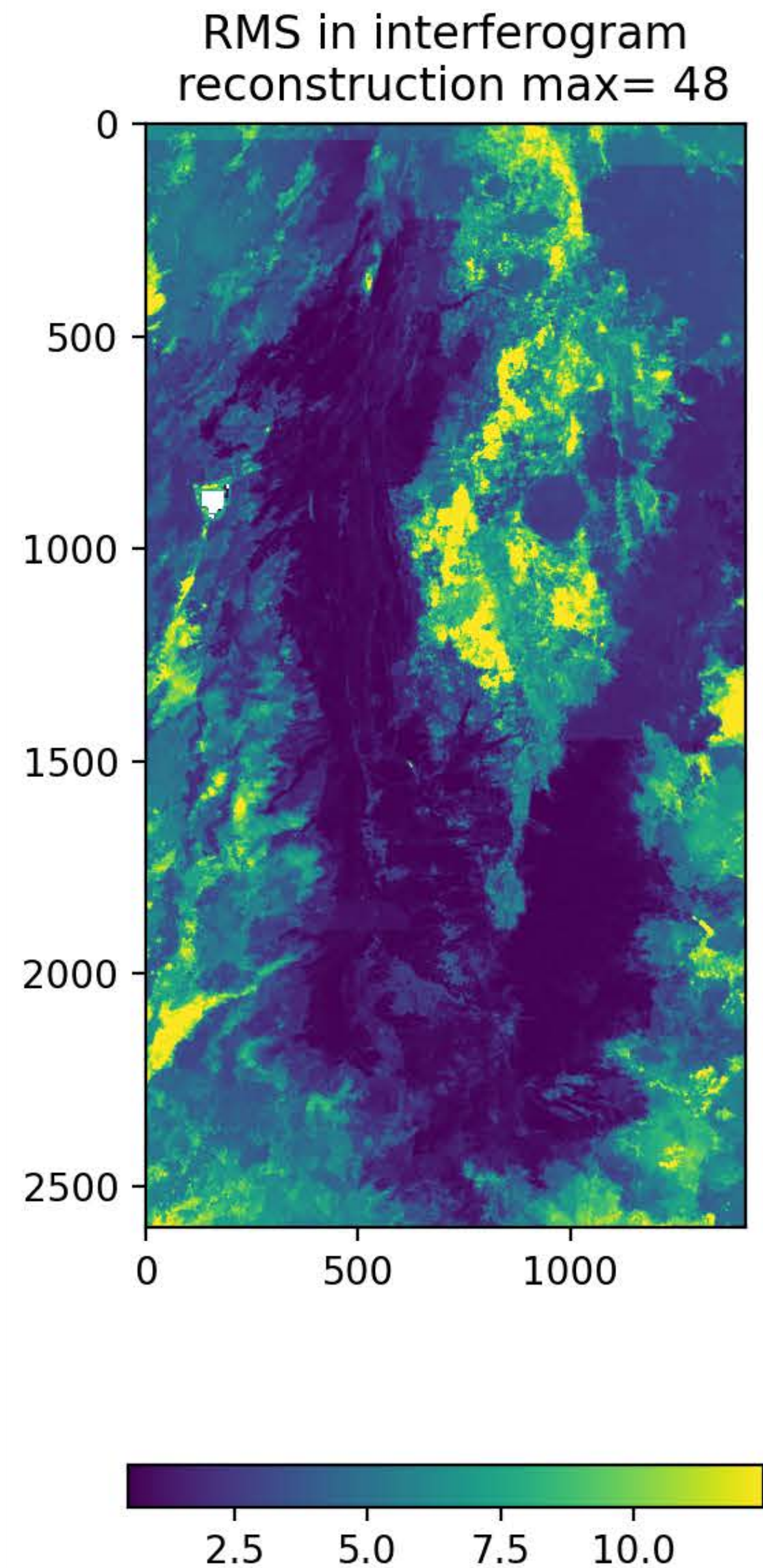
- Synthetic example with joint estimation of ϕ_j and a_n
- The state « learns » from the new data in the analysis step

$$\text{state} = \text{forecast} + \text{gain} \times \text{innovation}$$

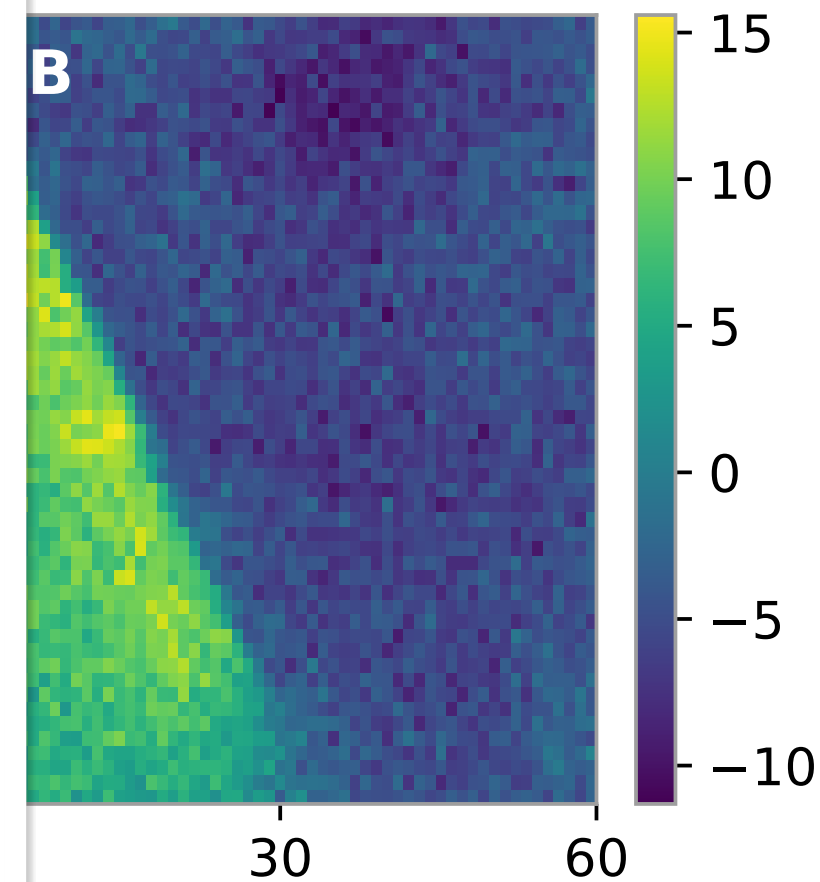
$$P_k^f H_k^T \left(R_k + H_k P_k^f H_k^T \right)^{-1} \left(d_k - H_k m_k^f \right)$$



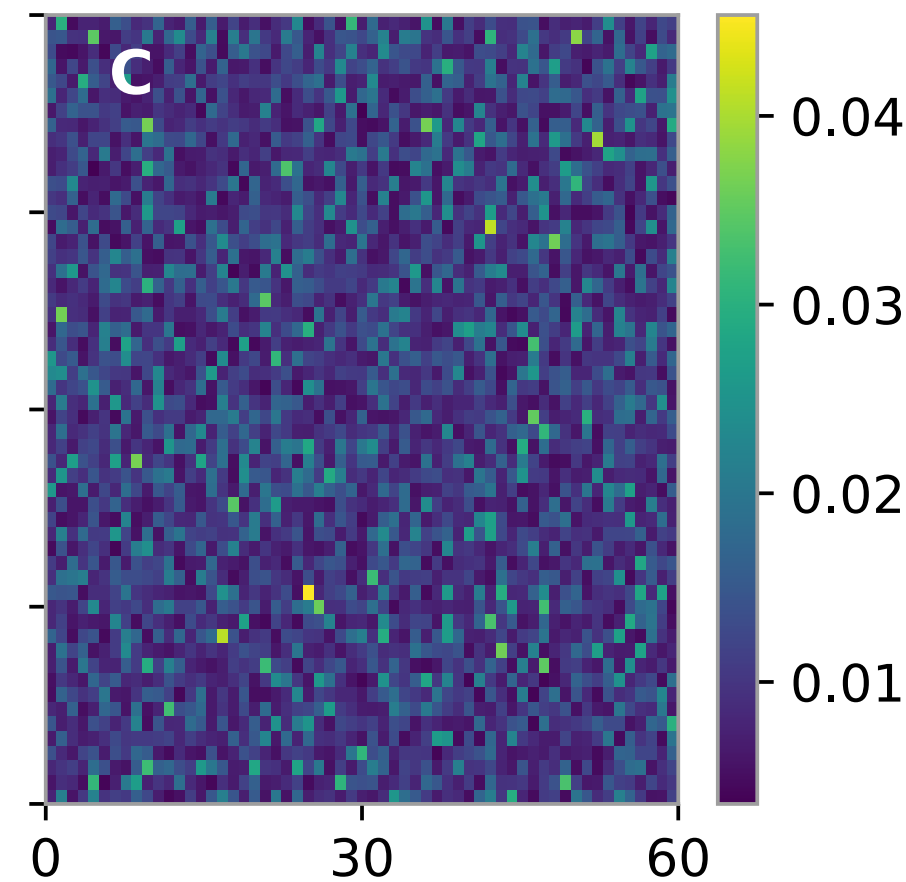
Tracking KFTS performance



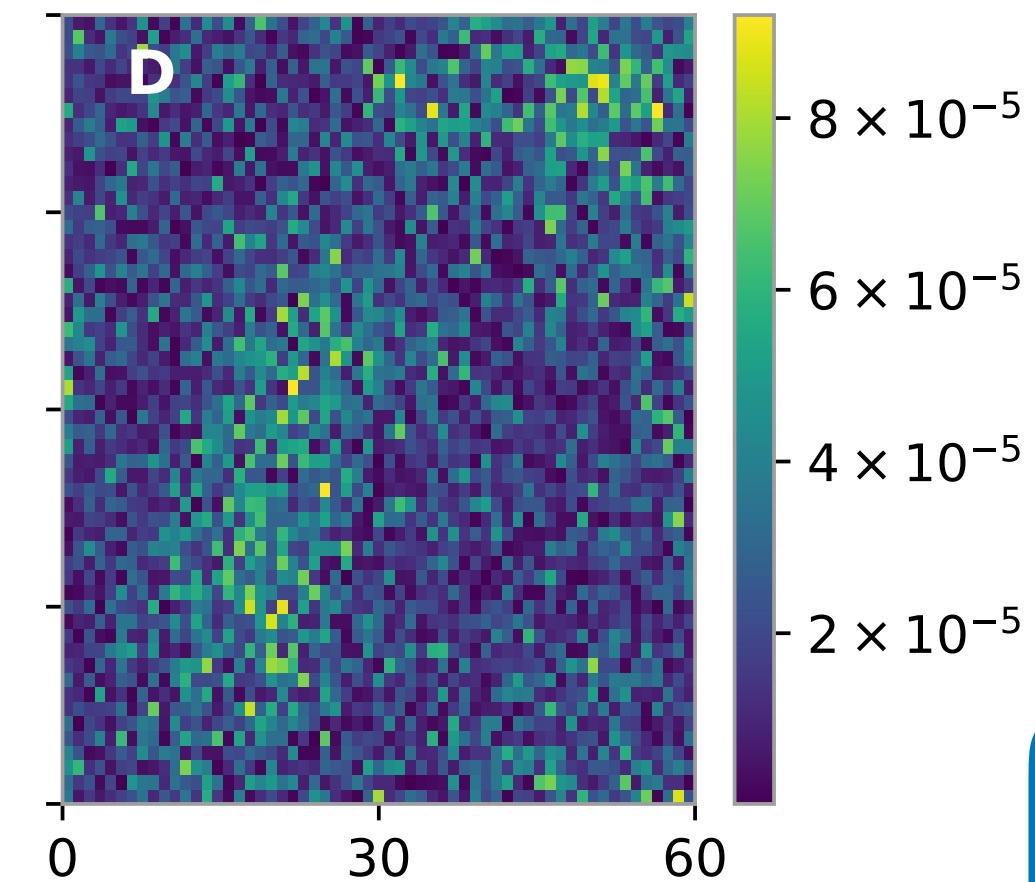
Reconstructed



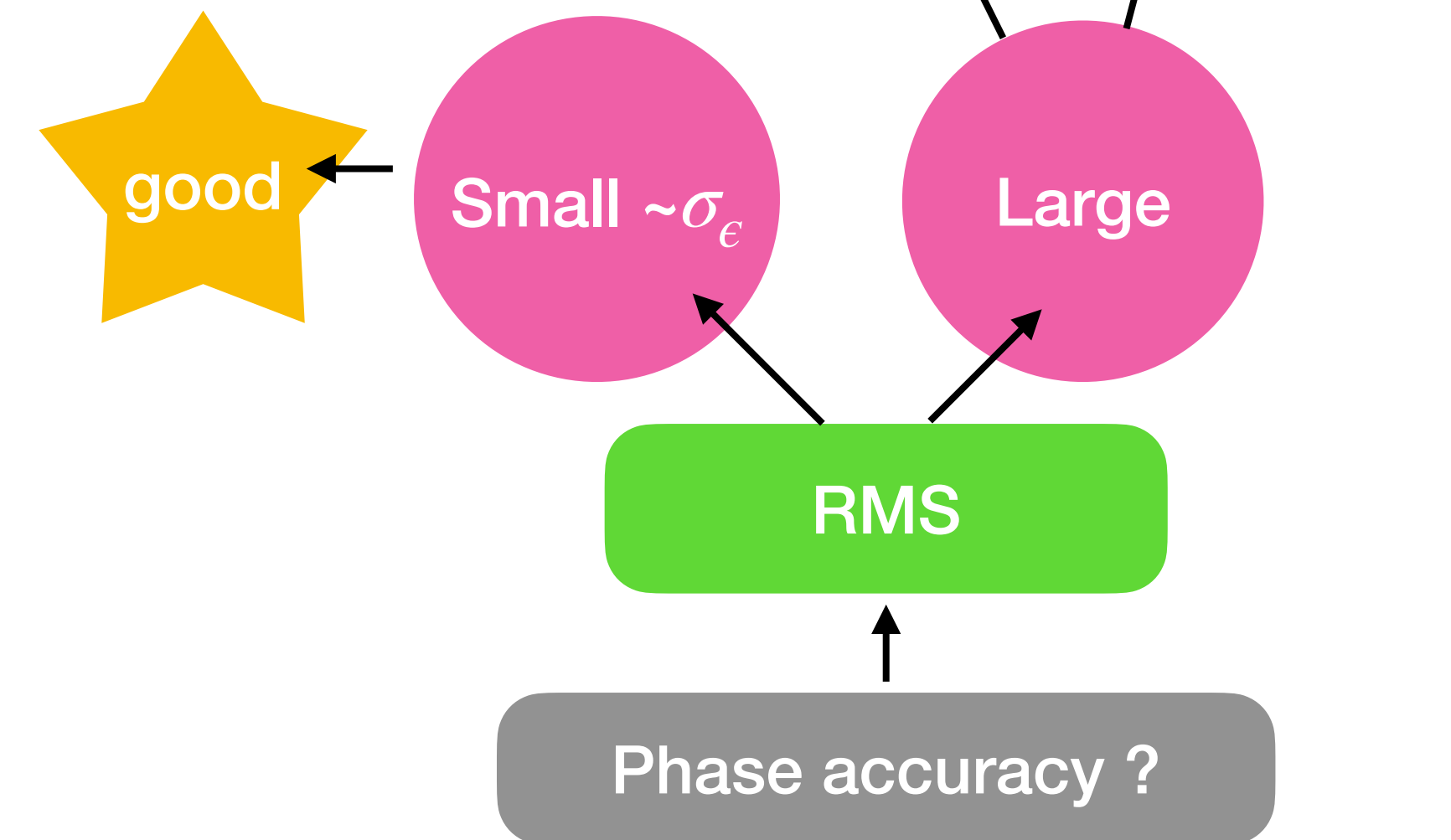
RMS(True,Recons.)



RMS(True,LSQ.)

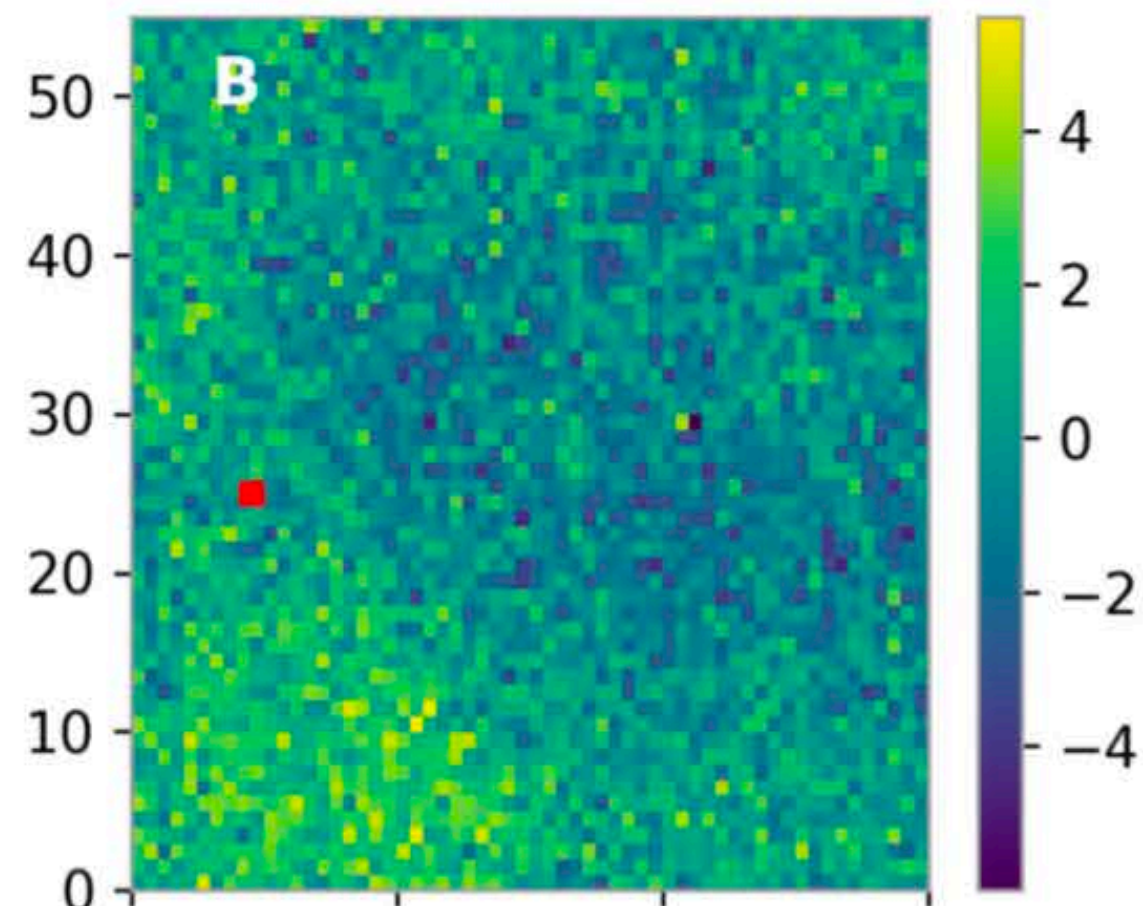


- 2D synthetic deformation field
- Fit to data > RMS
- Comparable to equivalent least-squares

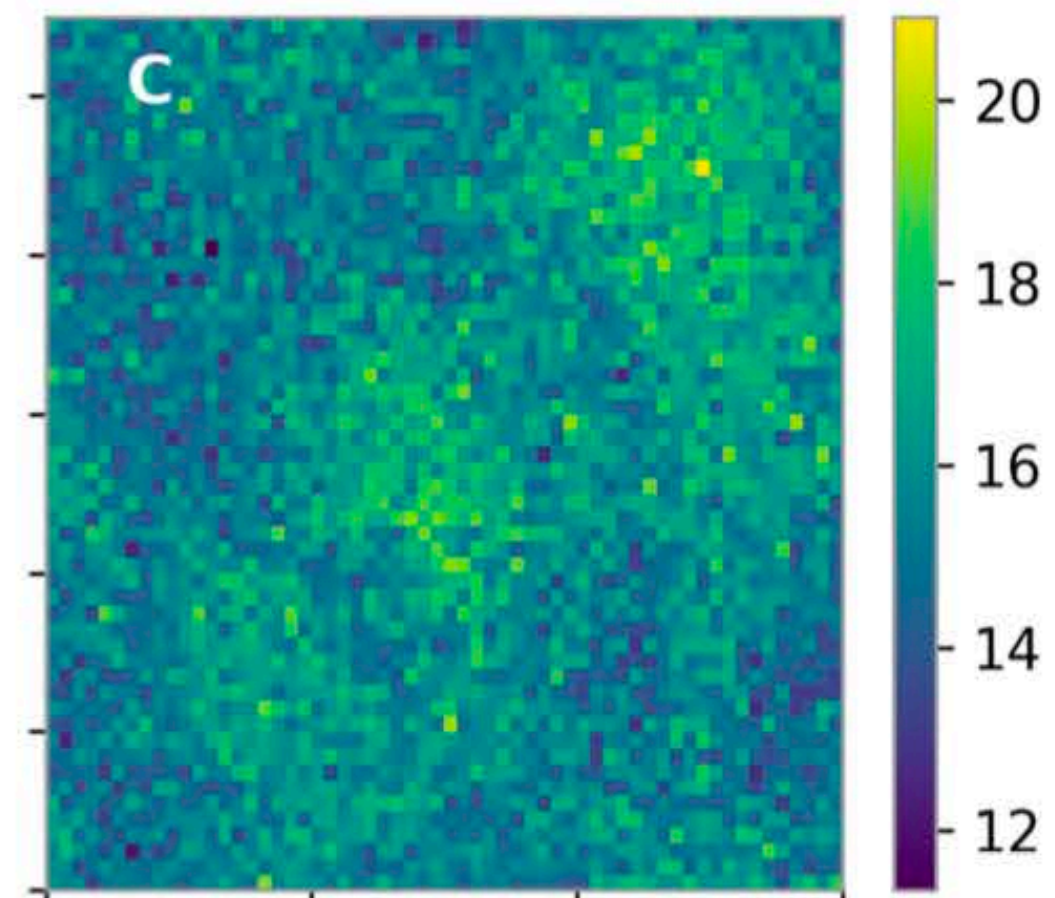


Tracking KFTS performance

Median



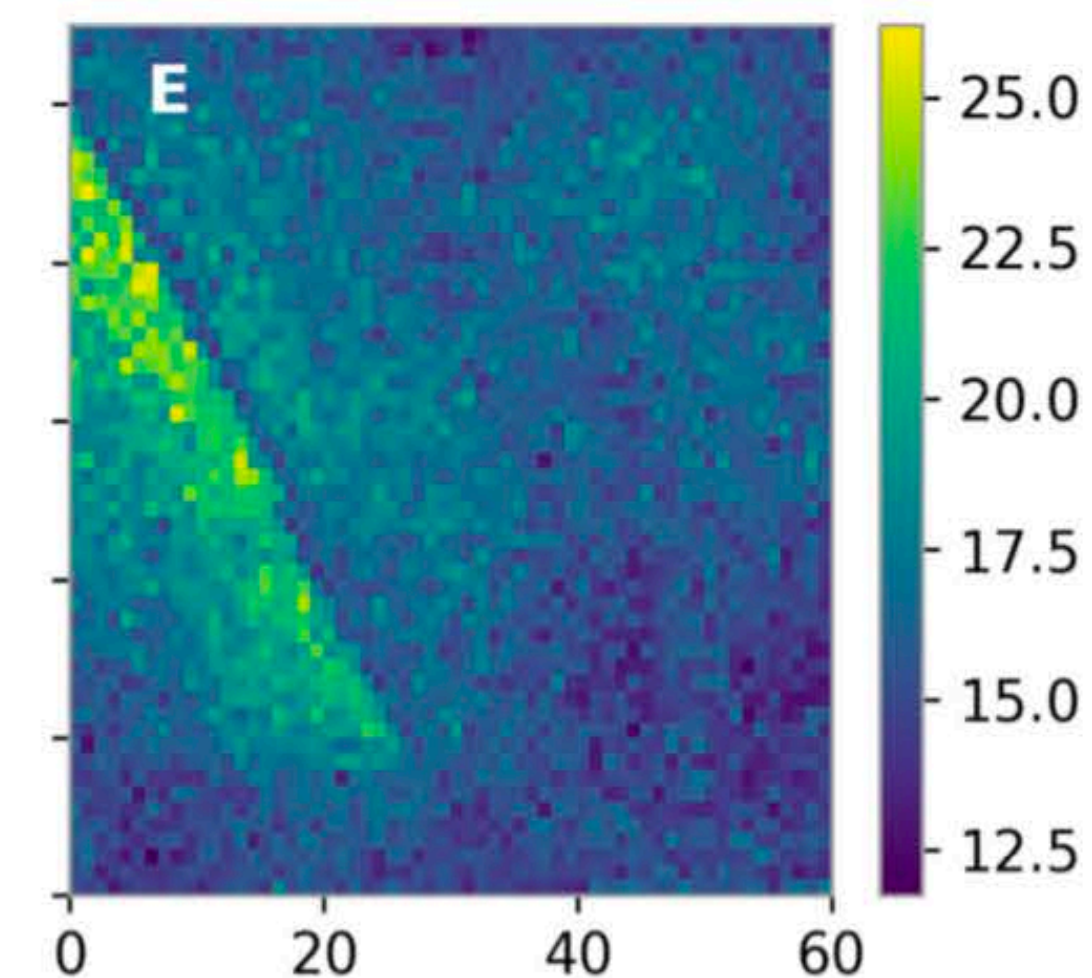
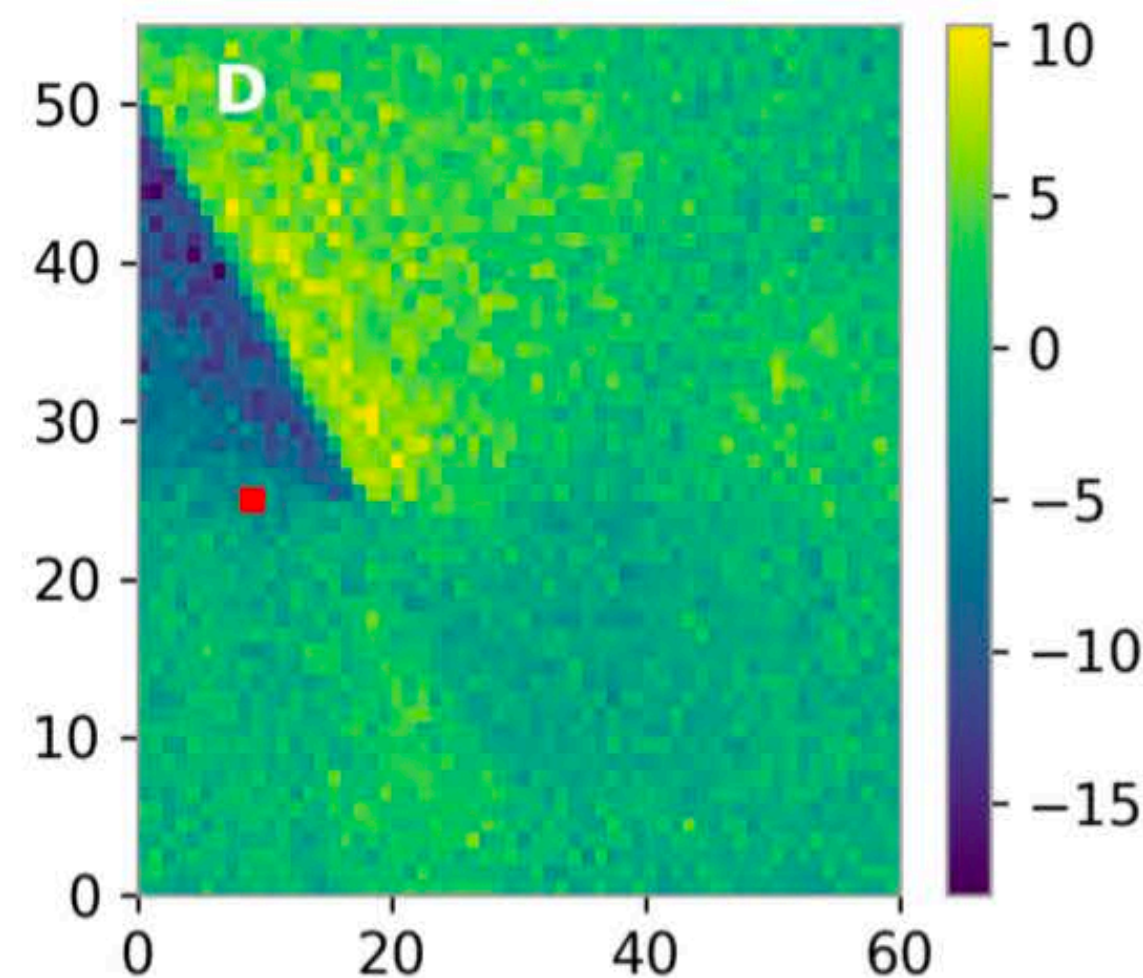
Standard dev



Innovation

Optimal
Parameterization

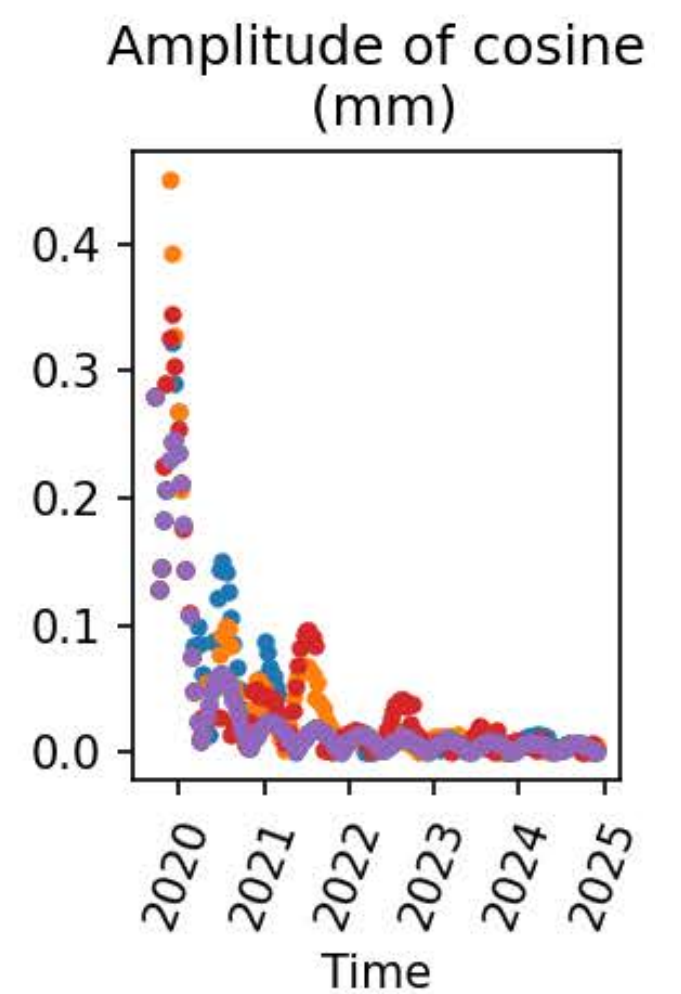
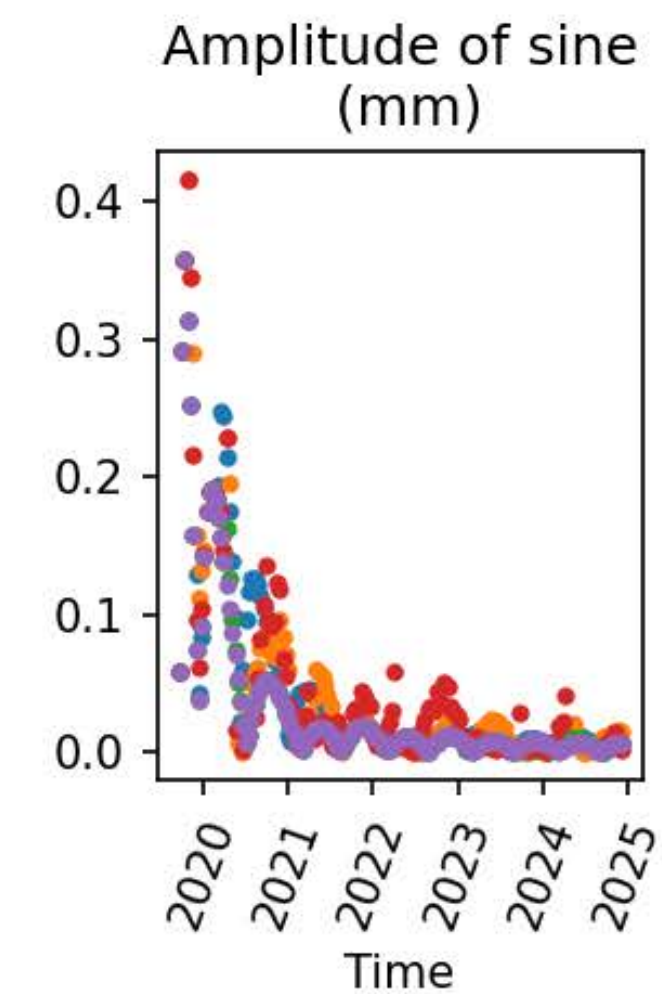
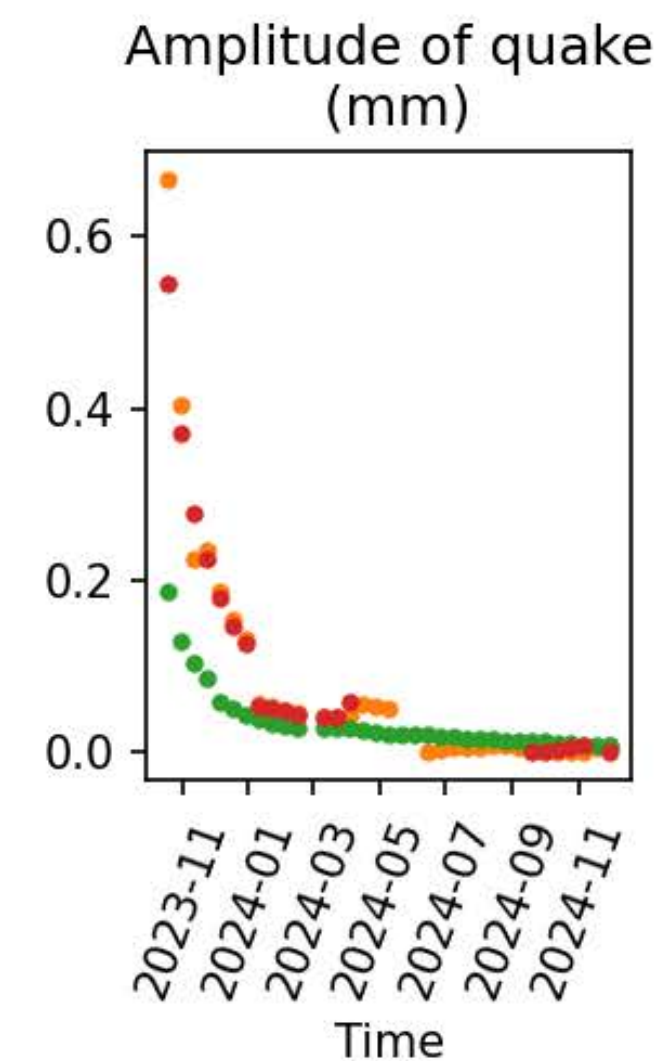
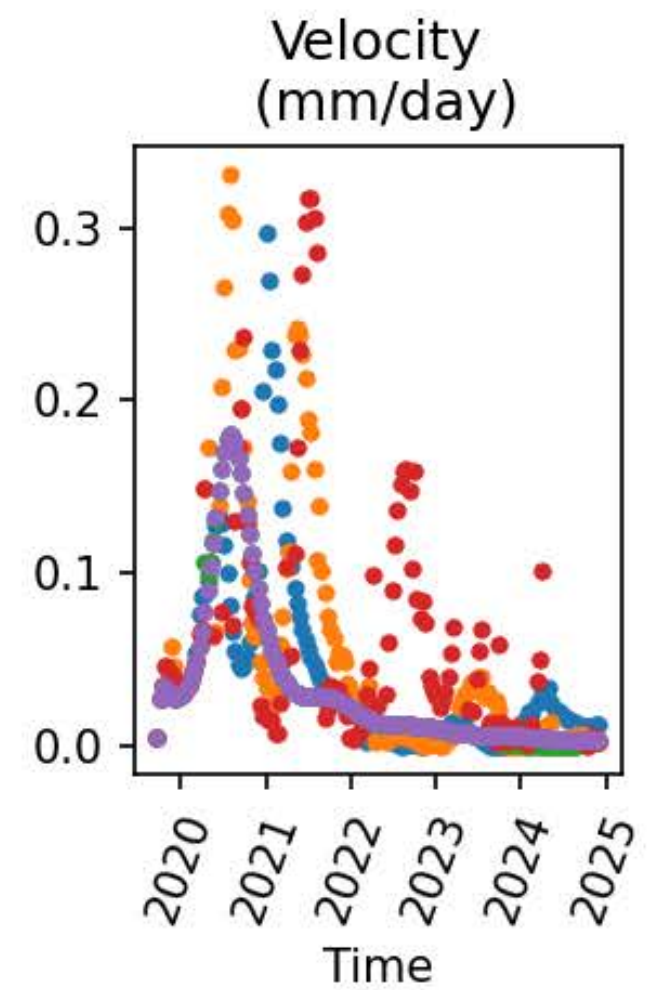
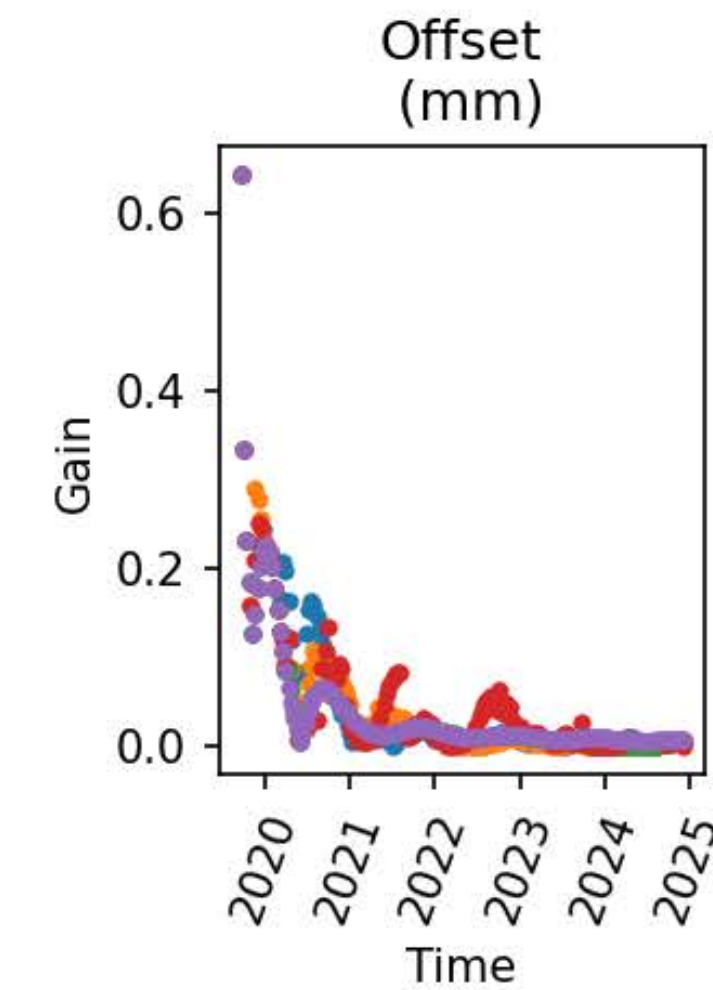
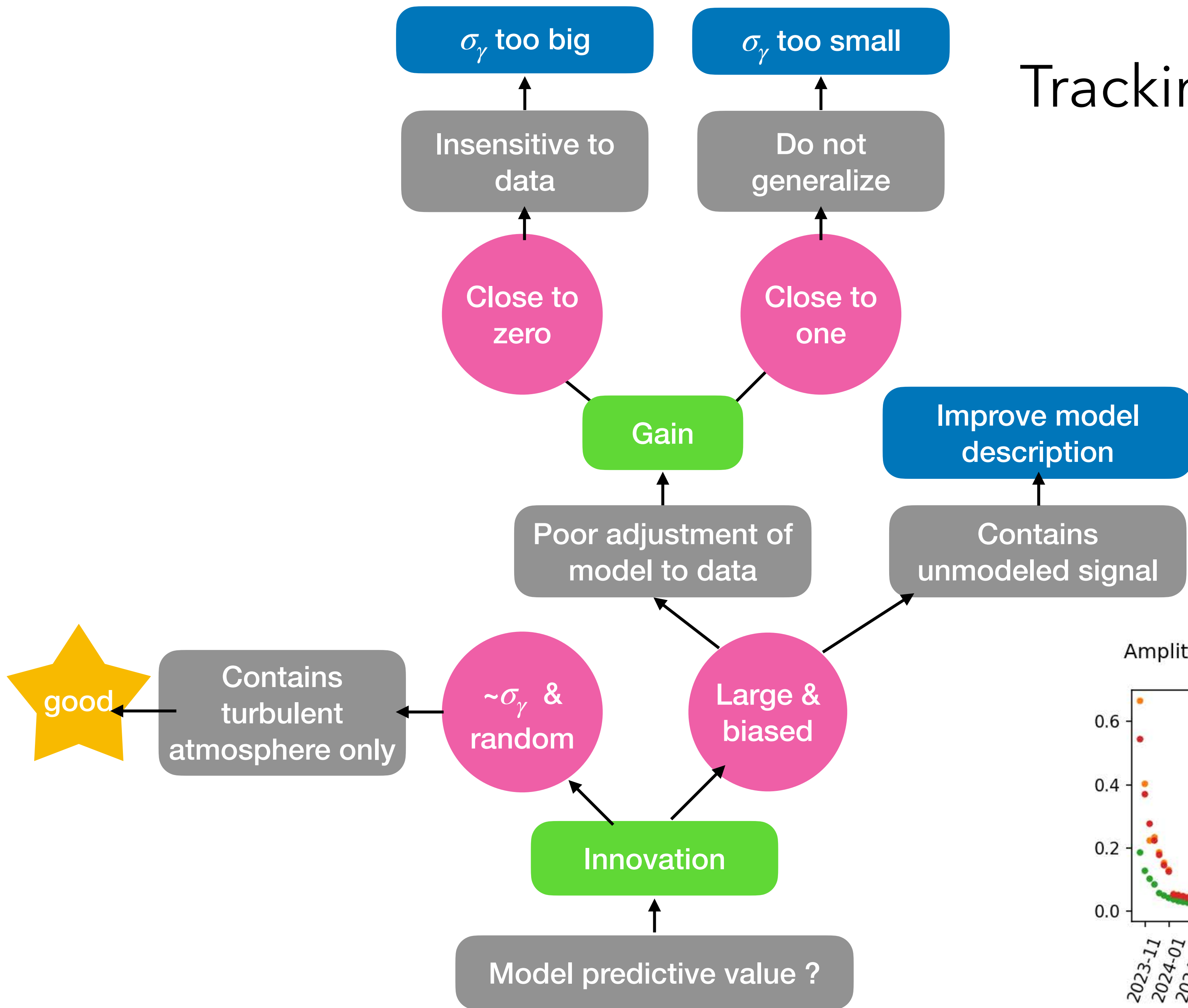
Missing
functional
elements
(H_{eq}, S_{sse})



$$\text{state} = \text{forecast} + \text{gain} \times \text{innovation}$$

$$P_k^f H_k^T \left(R_k + H_k P_k^f H_k^T \right)^{-1} \left(d_k - H_k m_k^f \right)$$

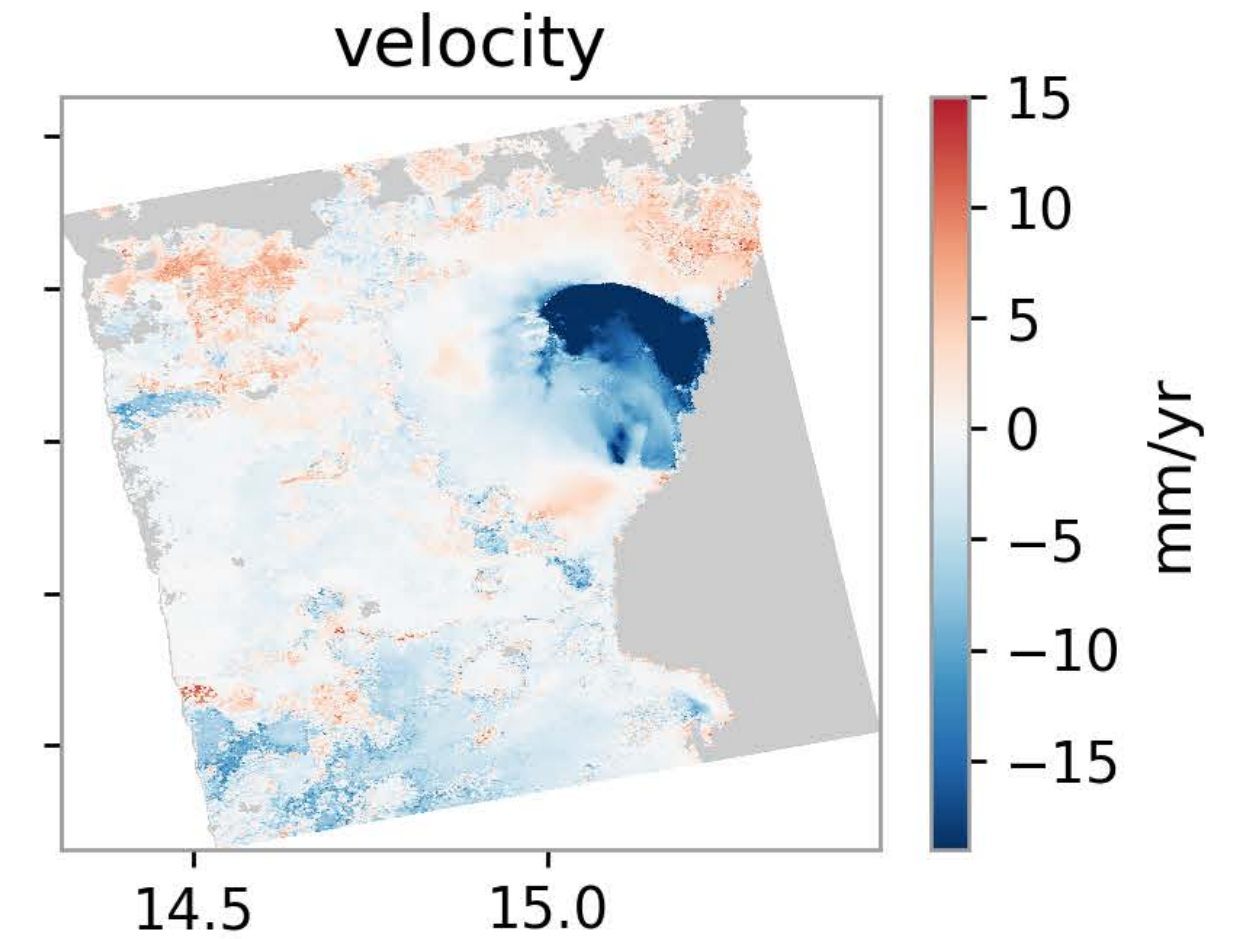
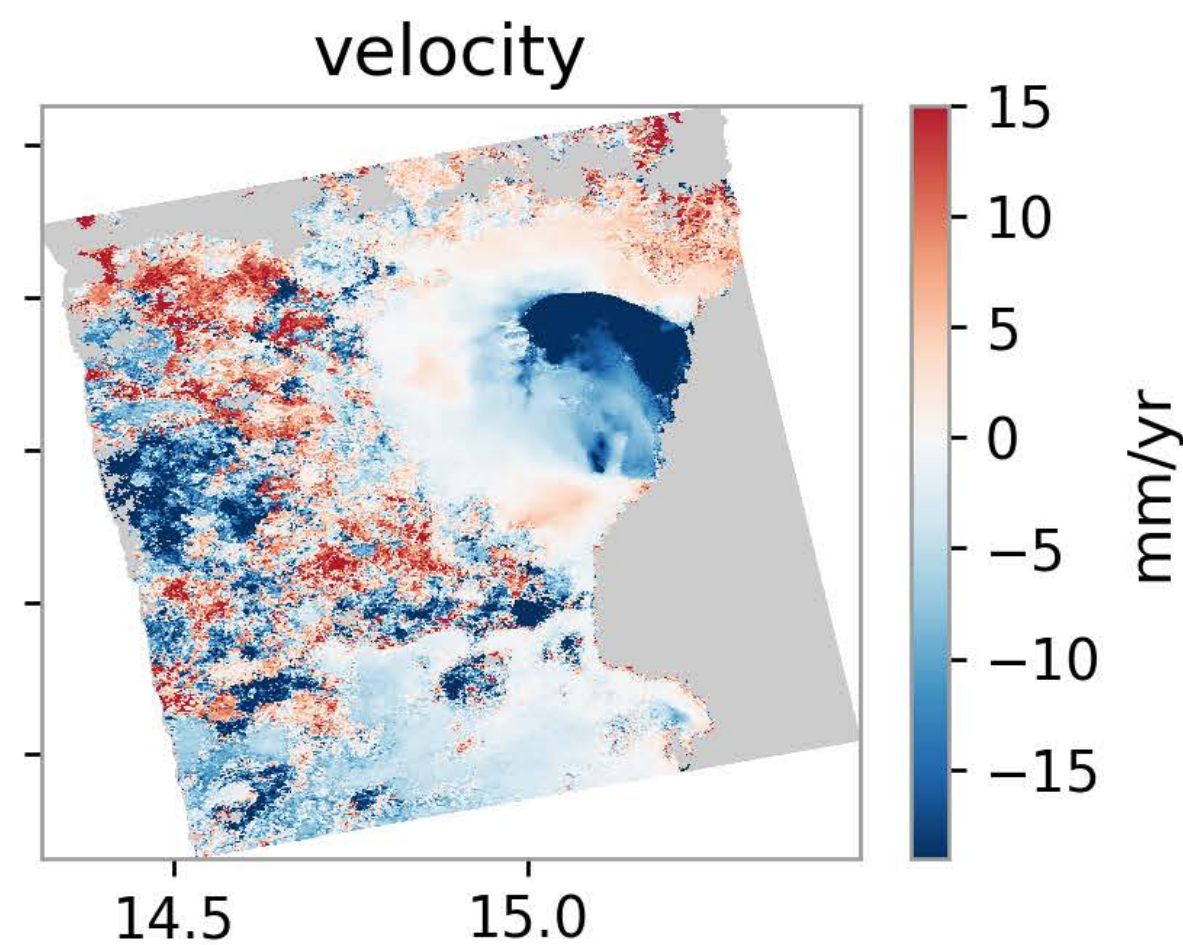
Tracking KFTS performance



Error definition

- Depends on priority : here fitting the InSAR observations over the model $\sigma_{\epsilon} \ll \sigma_{\gamma}$
- Can be empirically optimized using RMS, innovation and gain
- Can be constrained with more precise a priori knowledge

Over-trusting the data
(small σ_{ϵ})



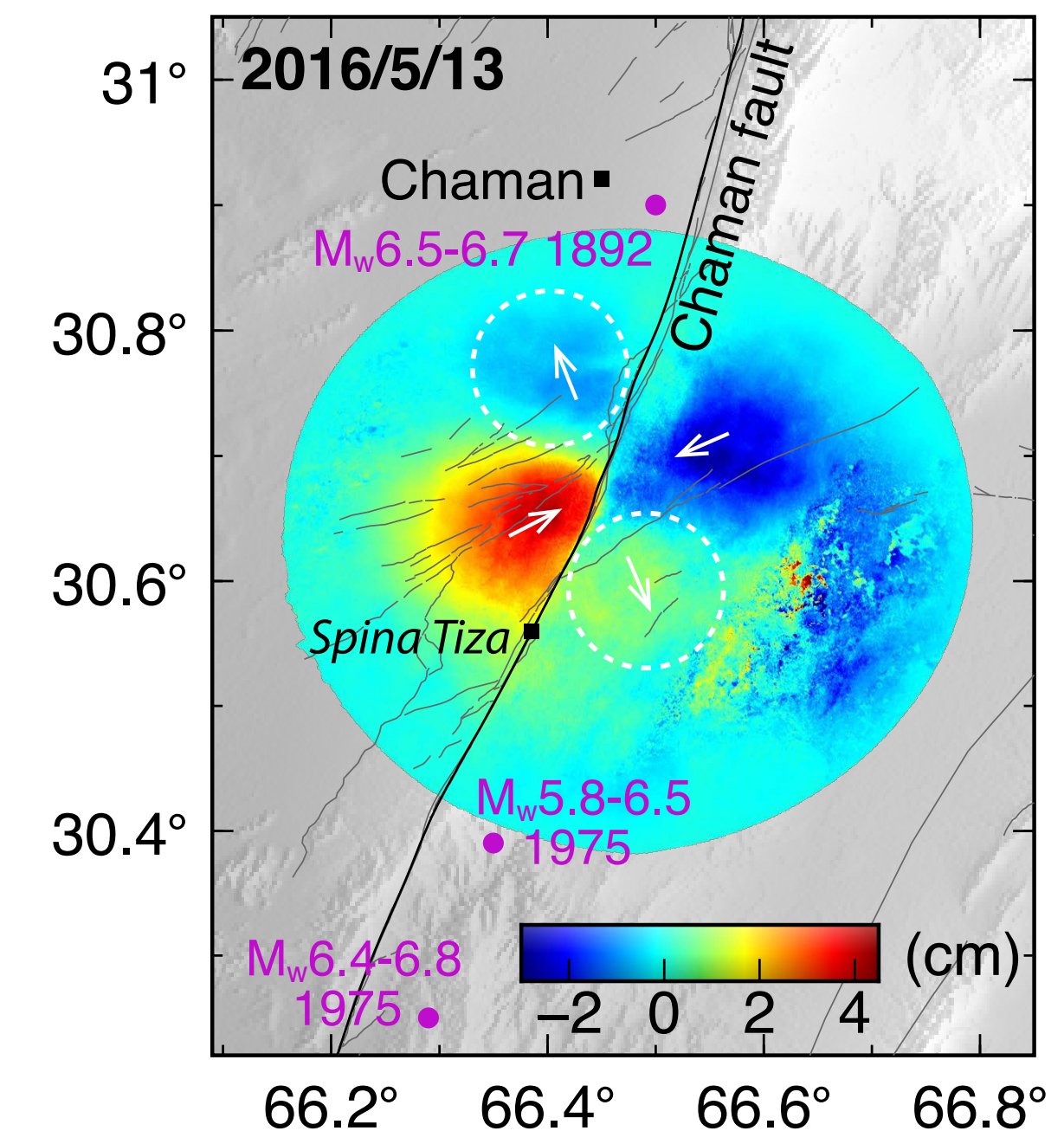
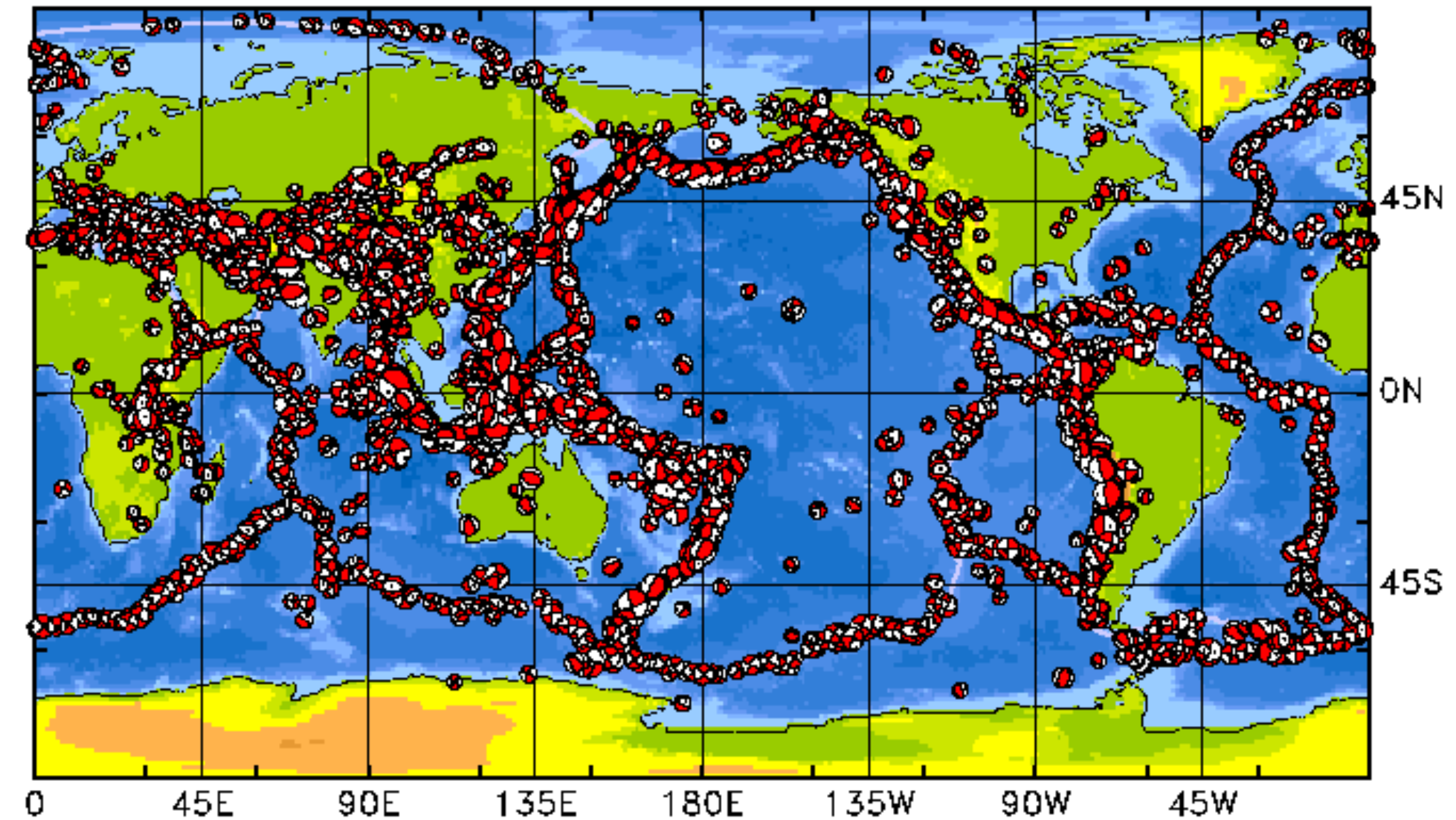
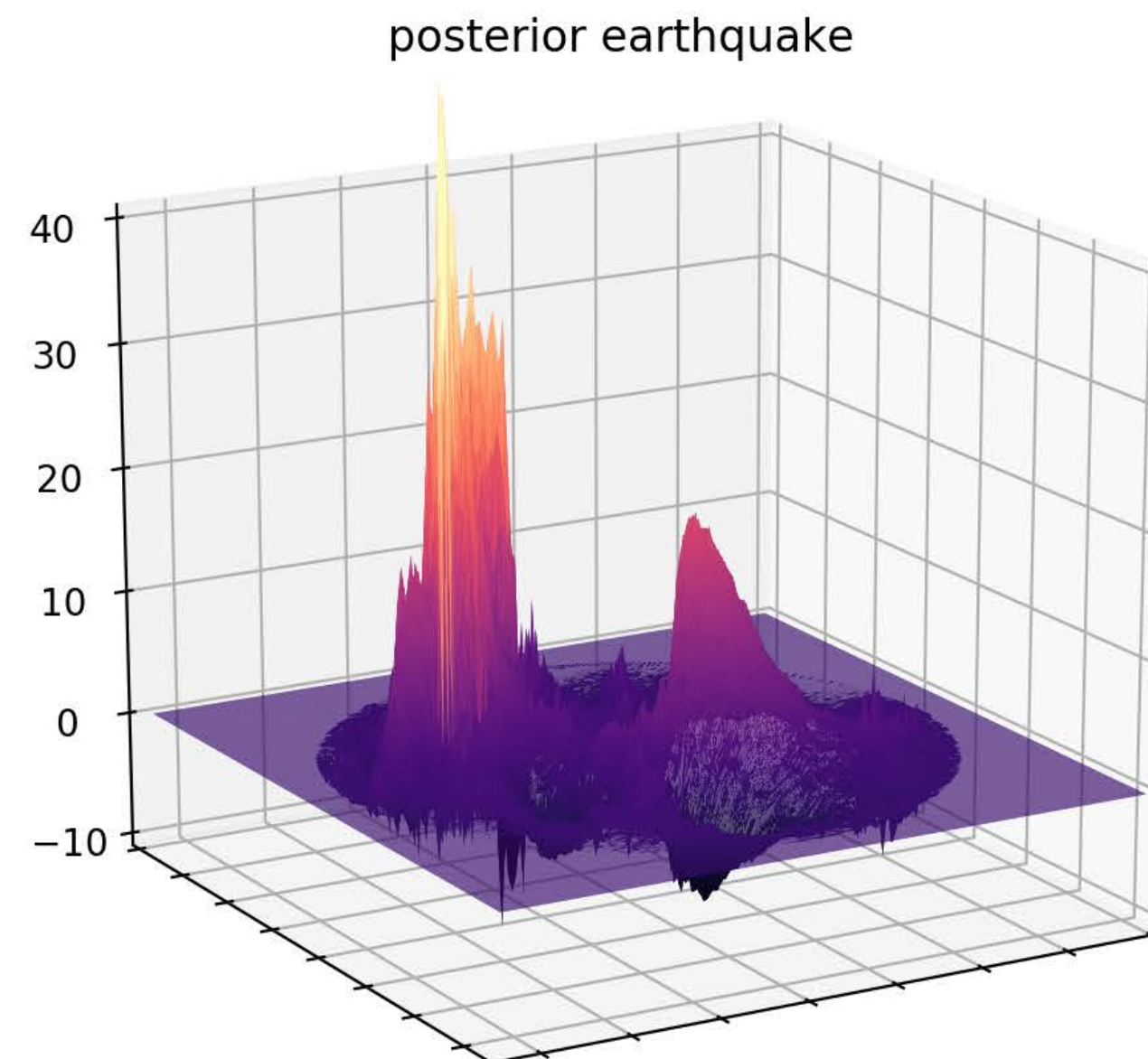
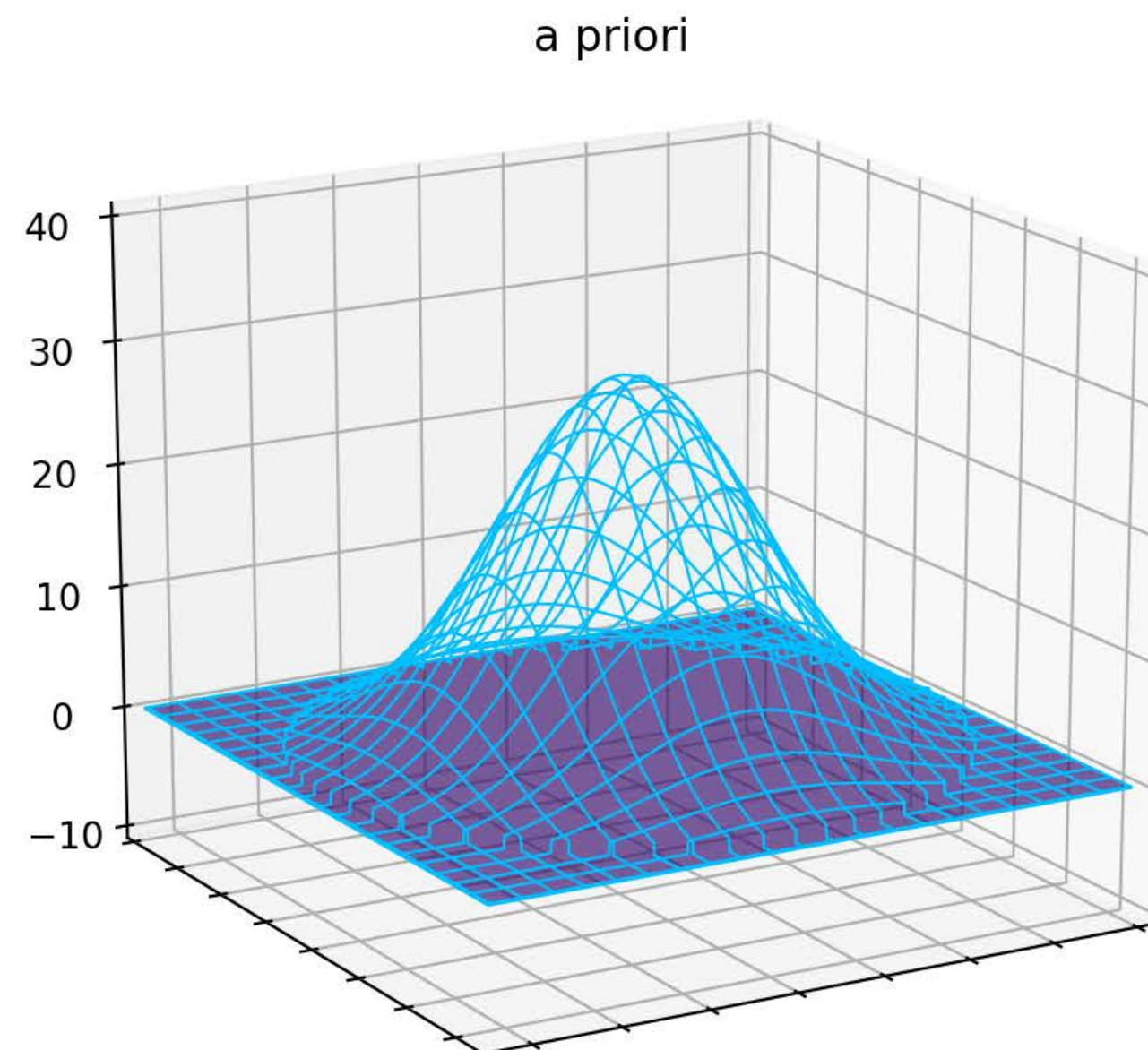
Prior knowledge definition

- May limit model parameters amplitudes and constrain them in space

 P_0 m_0

Standard deviation

Mean



Error propagation

Governing equations

Data (Interferograms)

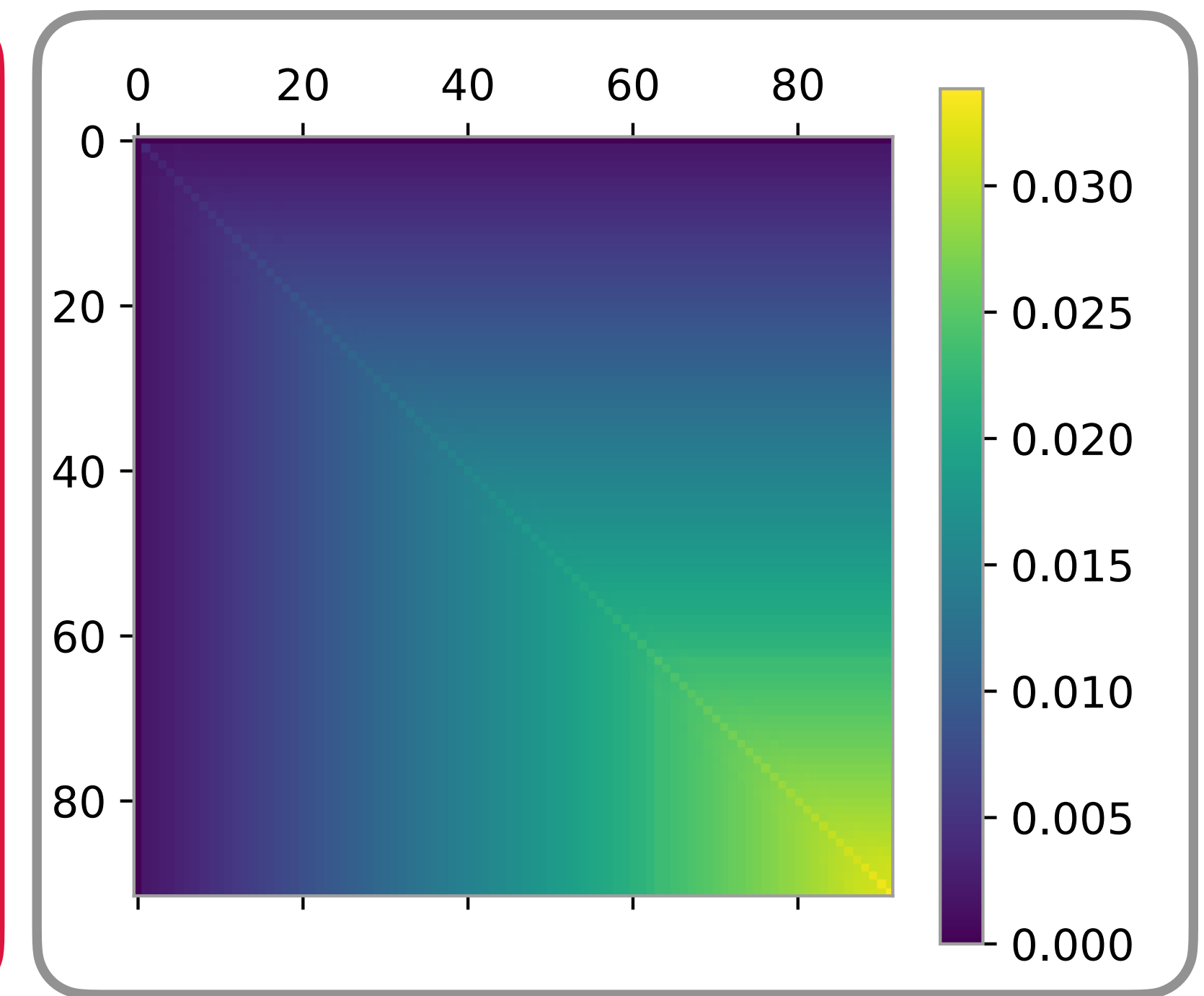
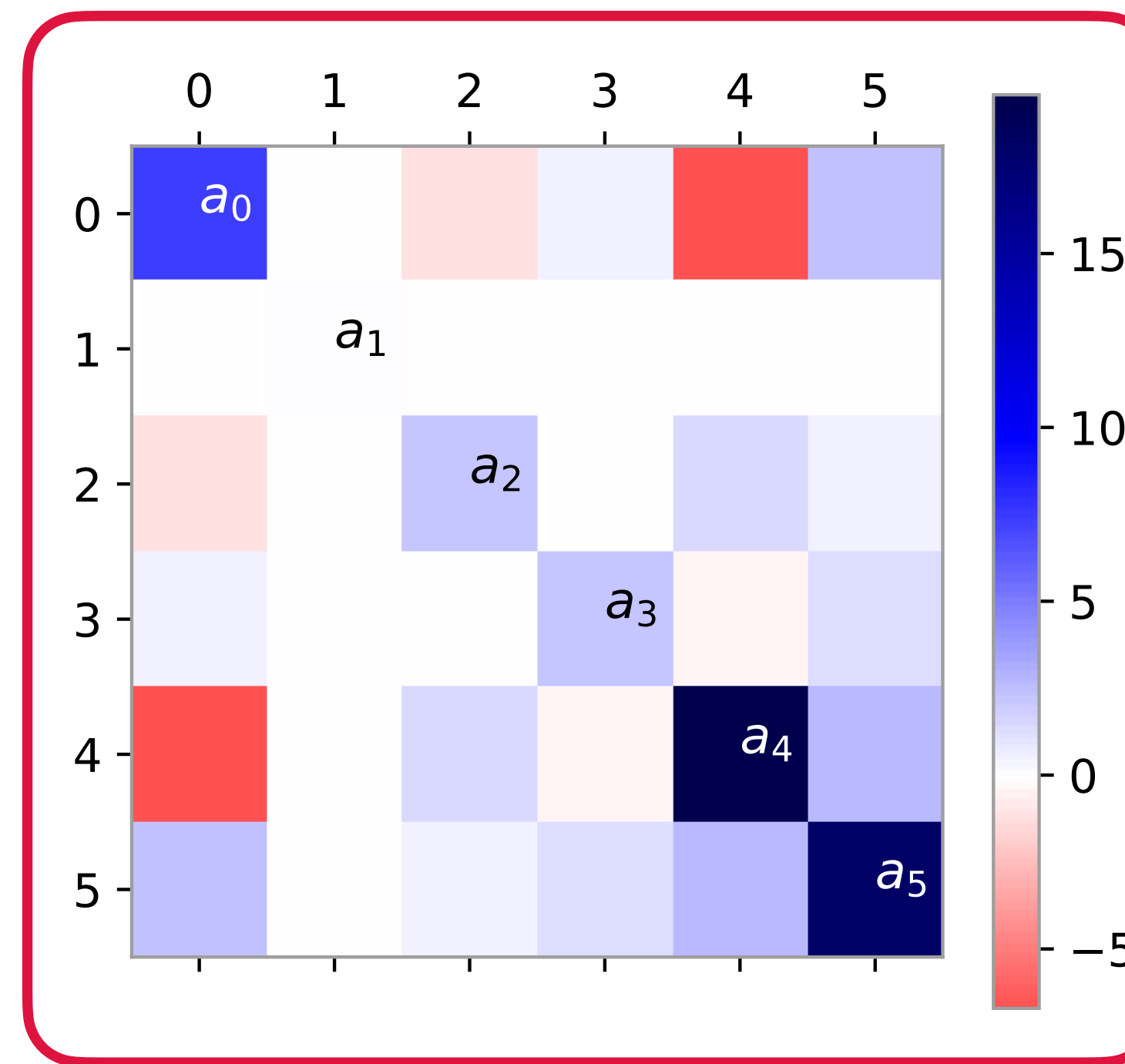
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error

Mismodeling

$$\phi(t_k) = \sum_{n=1}^N a_n f_n(t_k) + \gamma$$

Trajectory model
per pixel



Posterior Covariance

- Evolution of deformation relative to the first date → error accumulation as the distance to the reference grows
- Random noise
- Systematic noise → « phase bias », « phase drift » or « fading signal »

Smoothness constraints

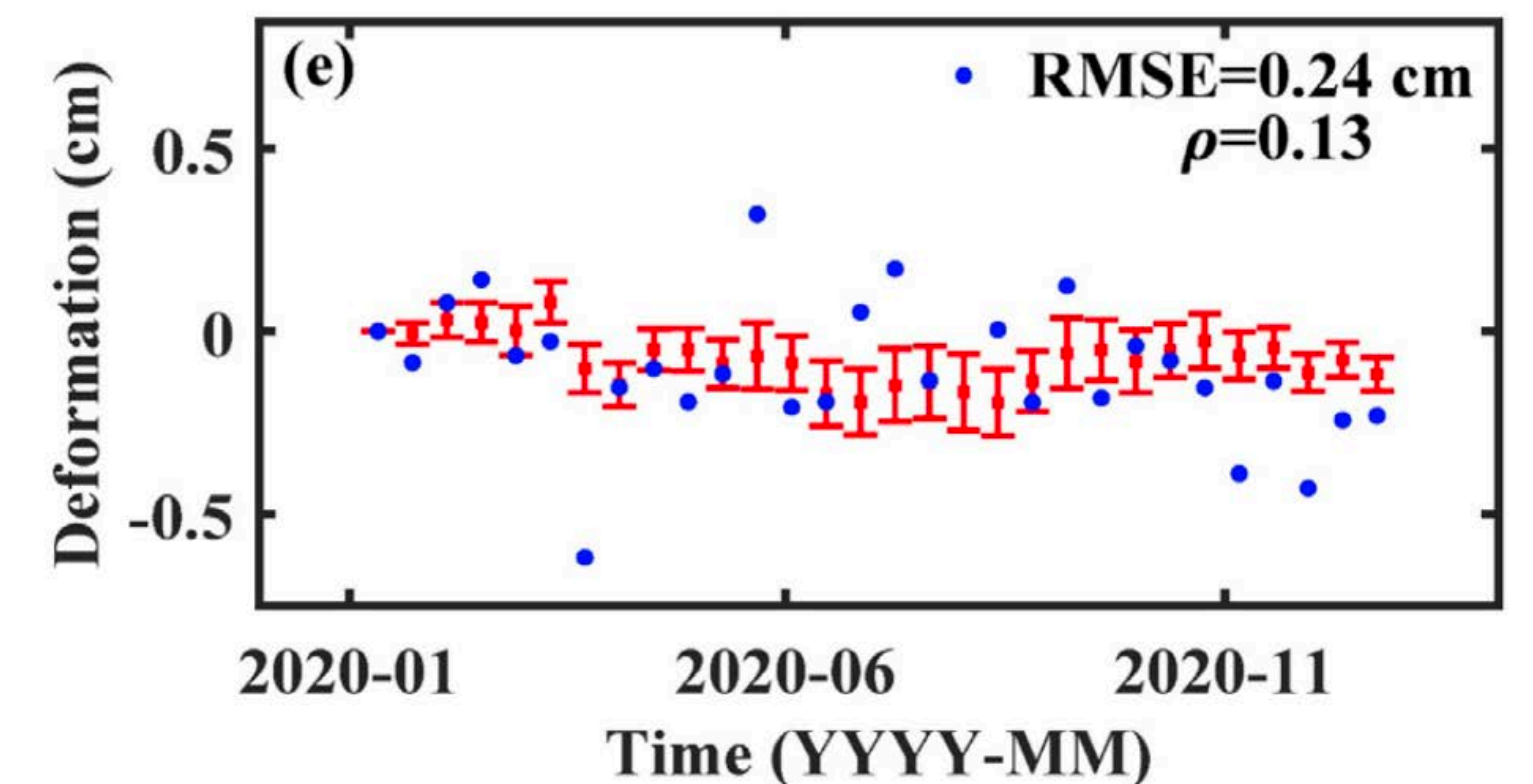
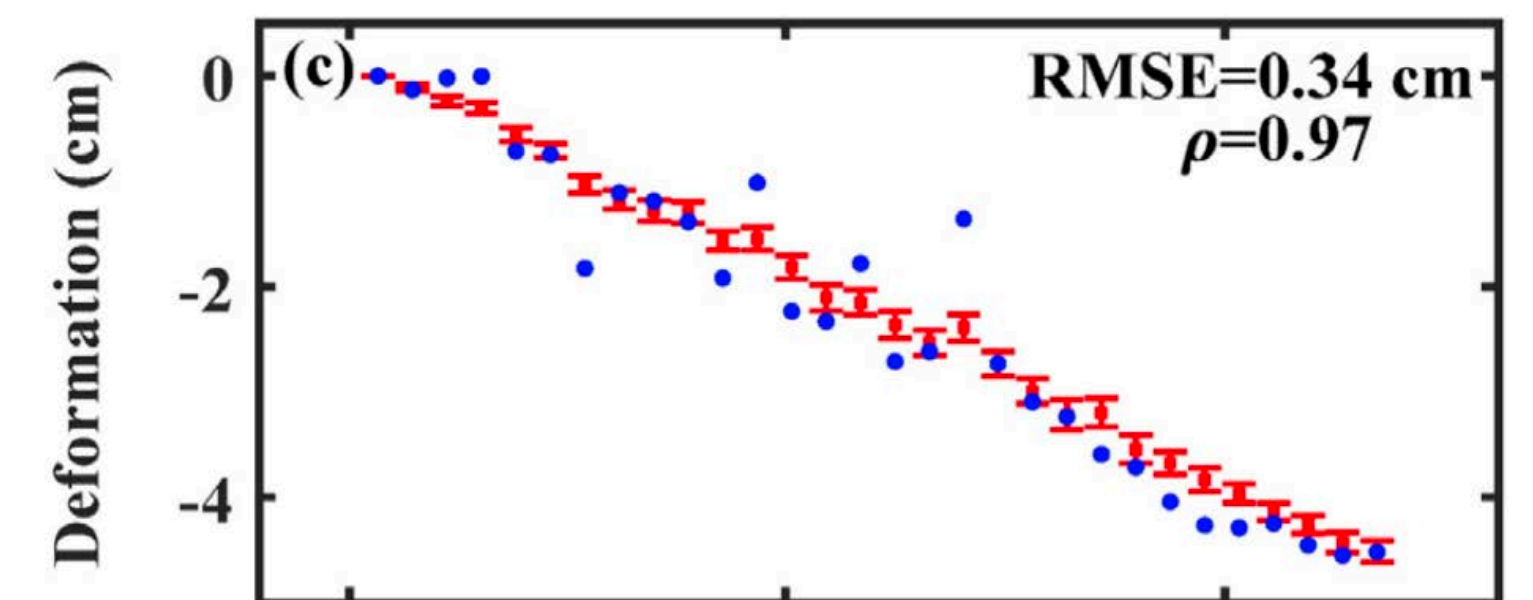
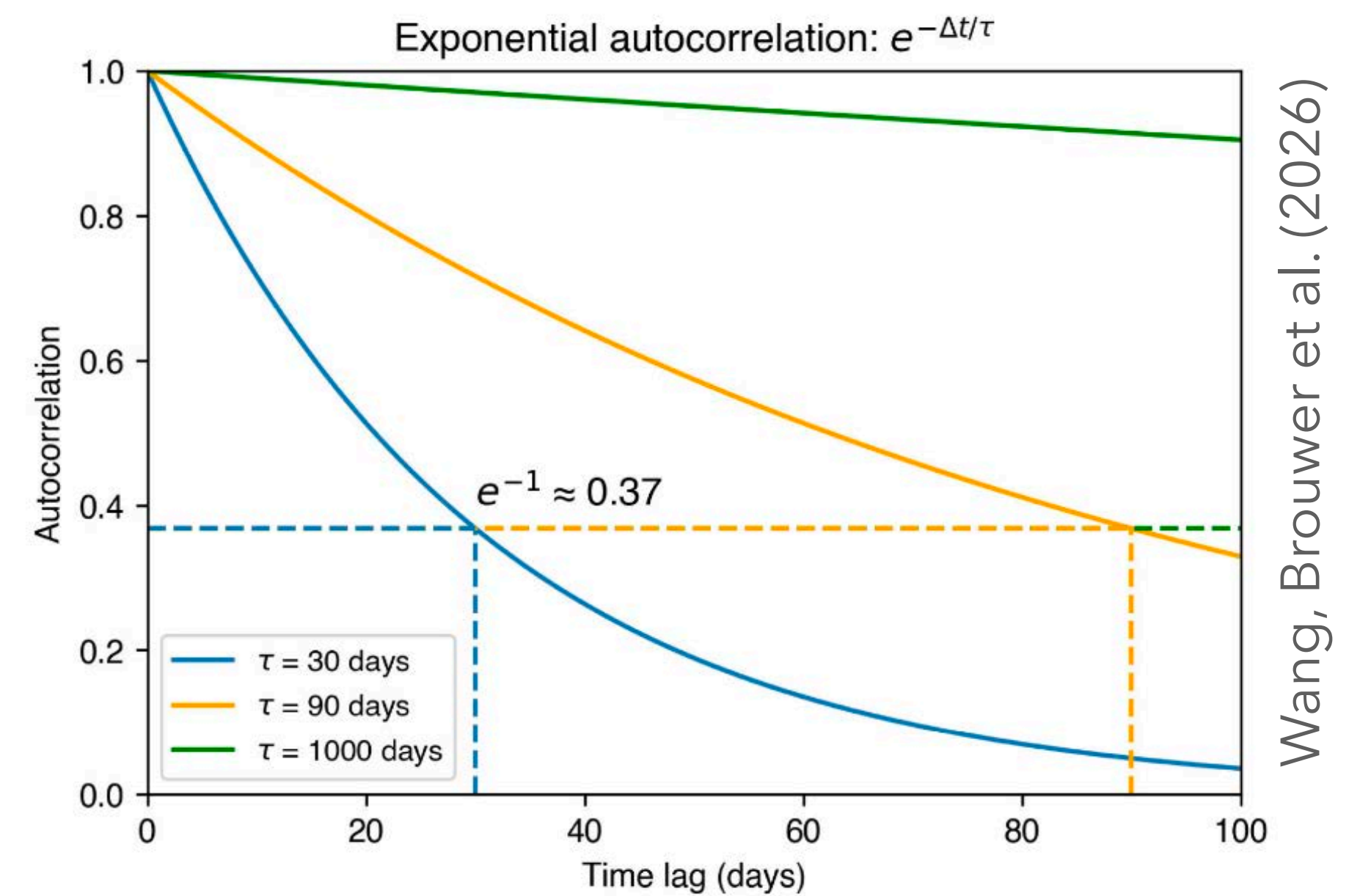
Temporal correlations ?

- Smoothness can be enforced defining correlations. If zero-mean Gaussian distribution can be in $\mathcal{C}_d$
- Larger data variance would reduce the data-noise impact on estimates

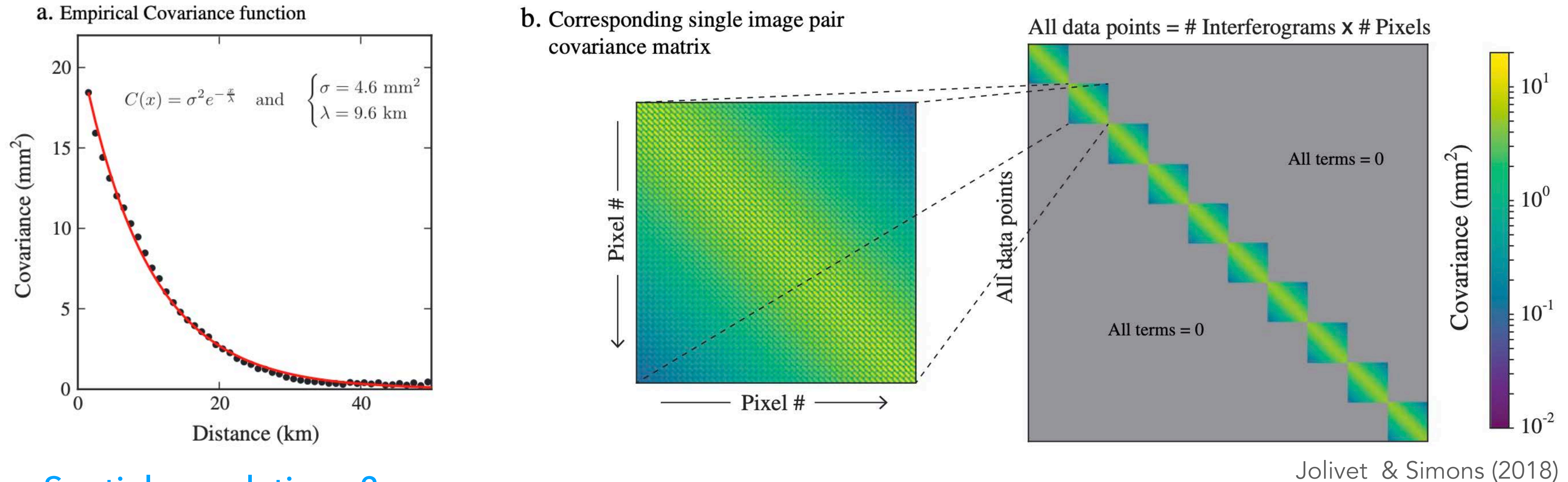
Spatial correlations ?

- None in KFTS as works pixel-wise

Learn noise statistics from prediction error using the Sage-Husa method



Smoothness constraints



Spatial correlations ?

- Pros : spatial correlation could help fill gaps which are only filled in time using the forecast...
- Cons : each pixel is a resolution cell with its own scattering mechanism including constructive and destructive interference (at the origin of speckle noise)
- In practice: rarely done in time series as numerically very expensive !

In this presentation

- A. Introduction to the design and parameterization of the Kalman Filter Time series analysis for InSAR
- B. Discussion on the complex nature of errors in InSAR phase change**
- C. Some examples of deformation monitoring
- D. Limits and conclusions

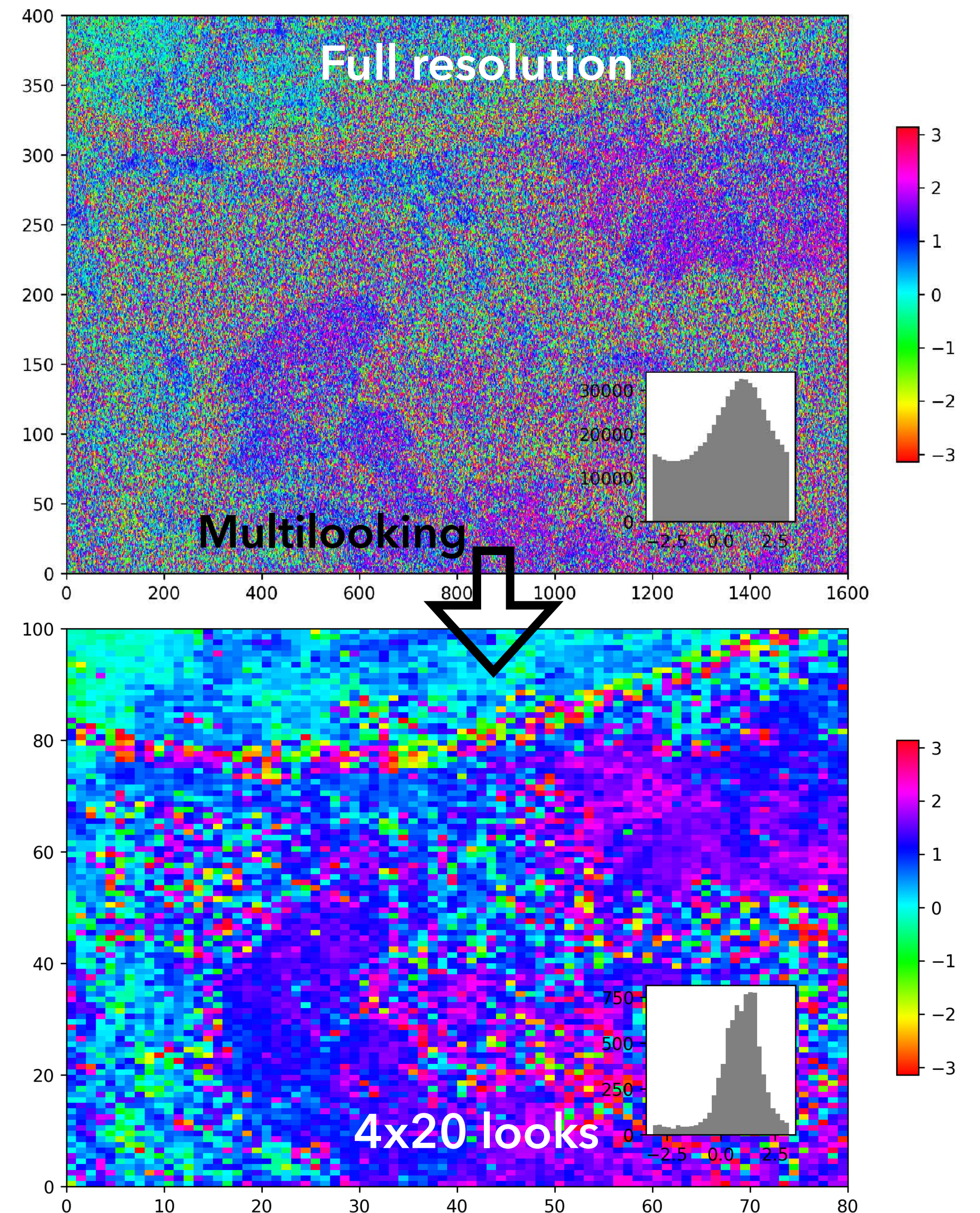
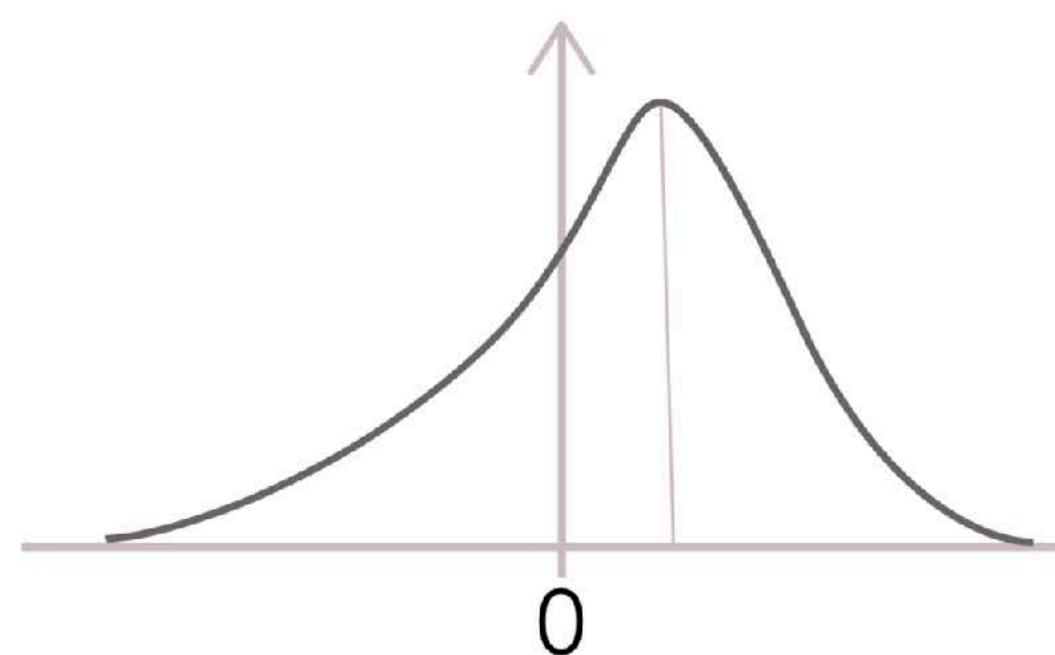
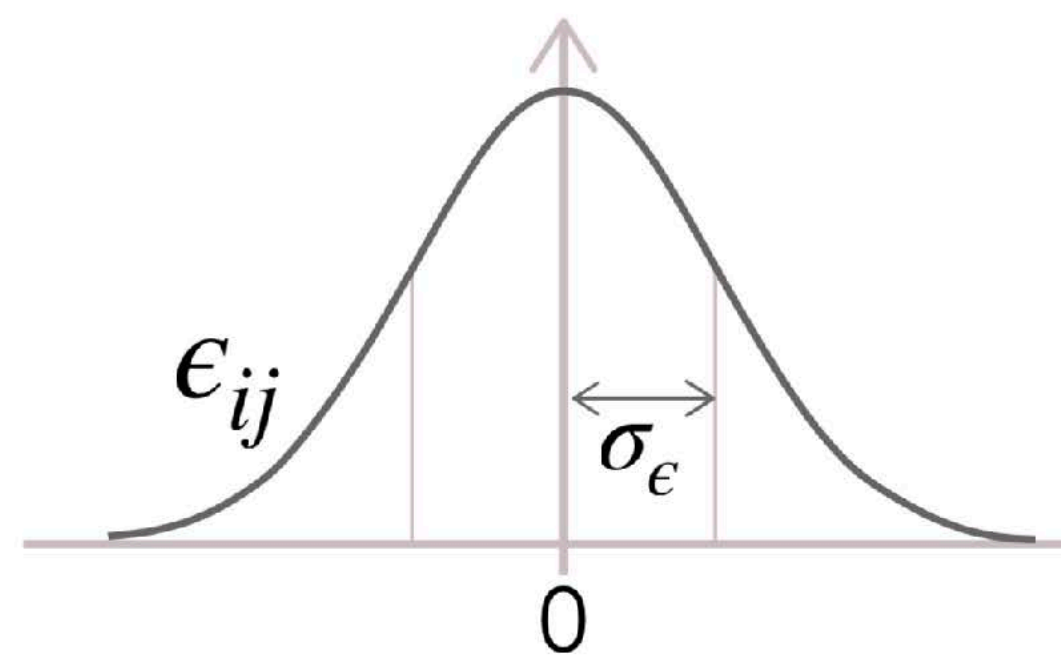
Refining the error model

Data (Interferograms)

$$\Phi_{ik} = \phi(t_k) - \phi(t_i) + \epsilon$$

error

- **Spatial averaging** on a number of pixels (or looks) of the complex image induces errors



Refining the error model

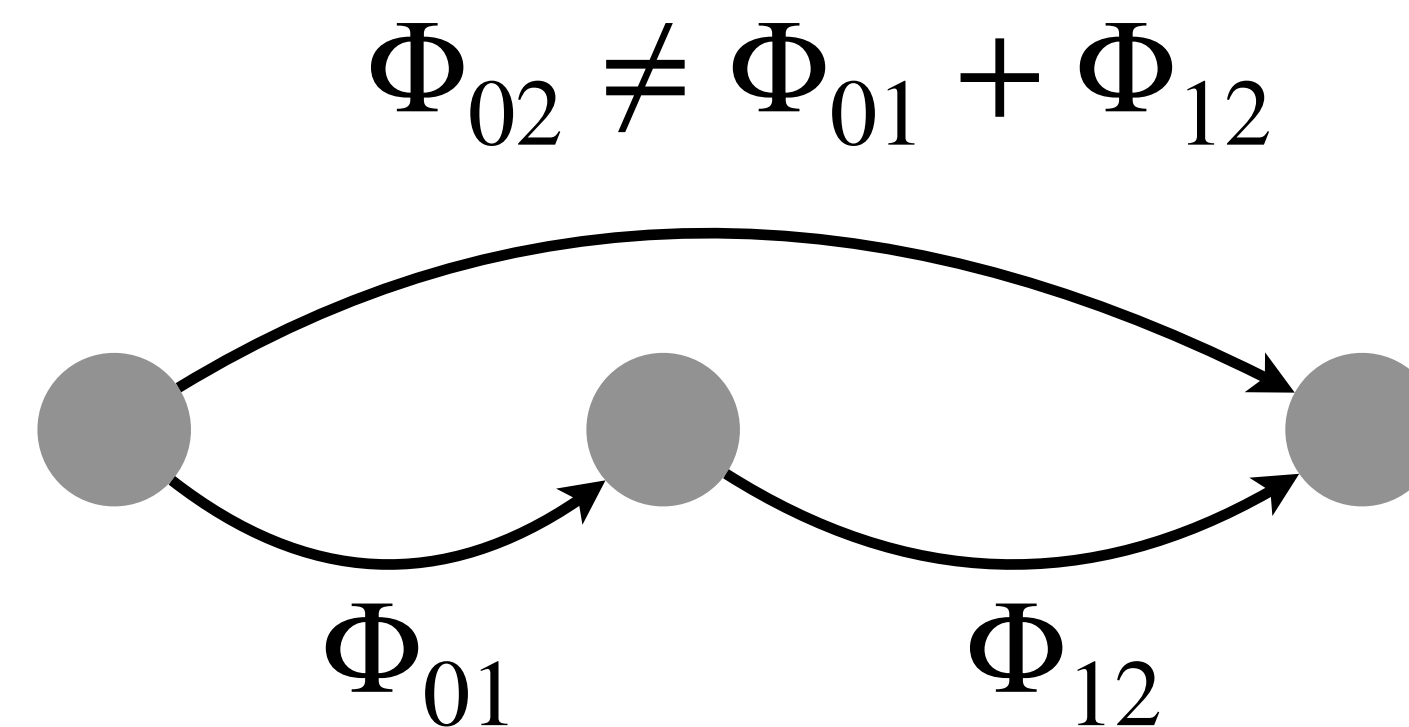
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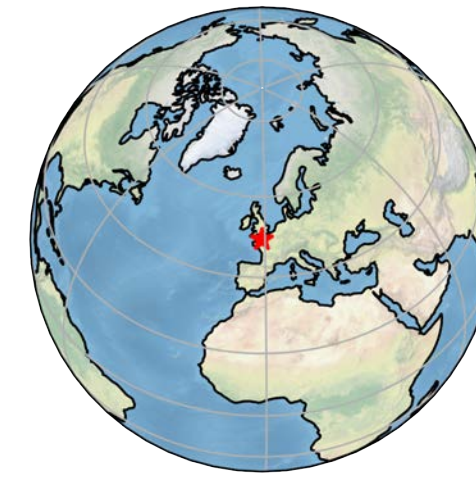
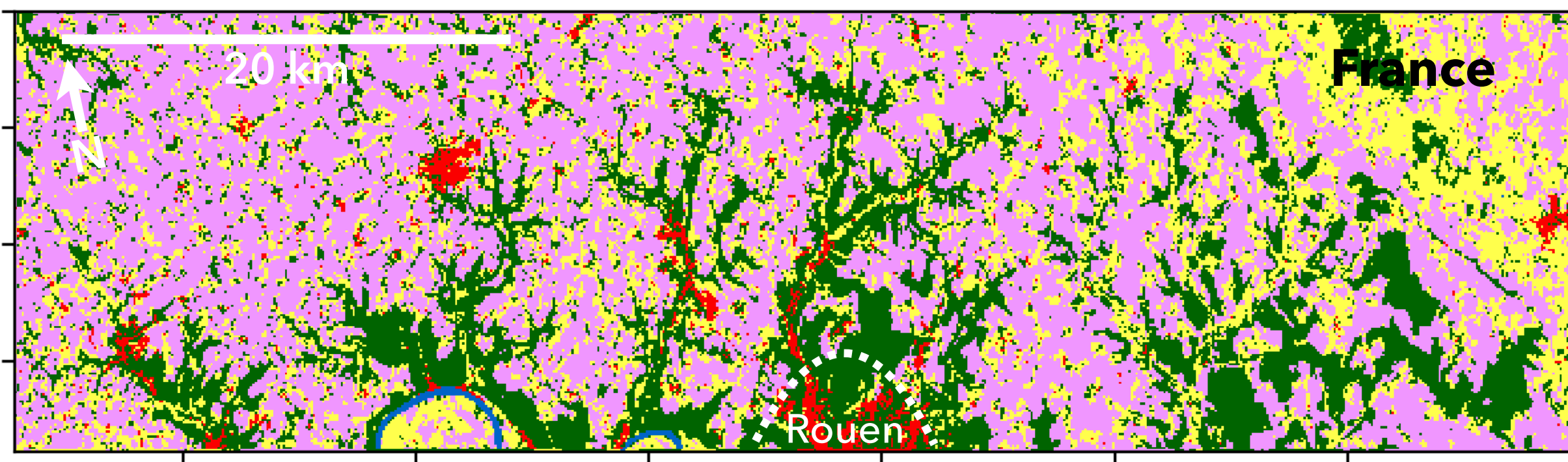
$$\Phi_{ik} = \phi(t_k) - \phi(t_i) + \epsilon$$

error

We study the distribution of **closure phase** $\Psi_{ijk} = \Phi_{ij} + \Phi_{jk} - \Phi_{ik}$

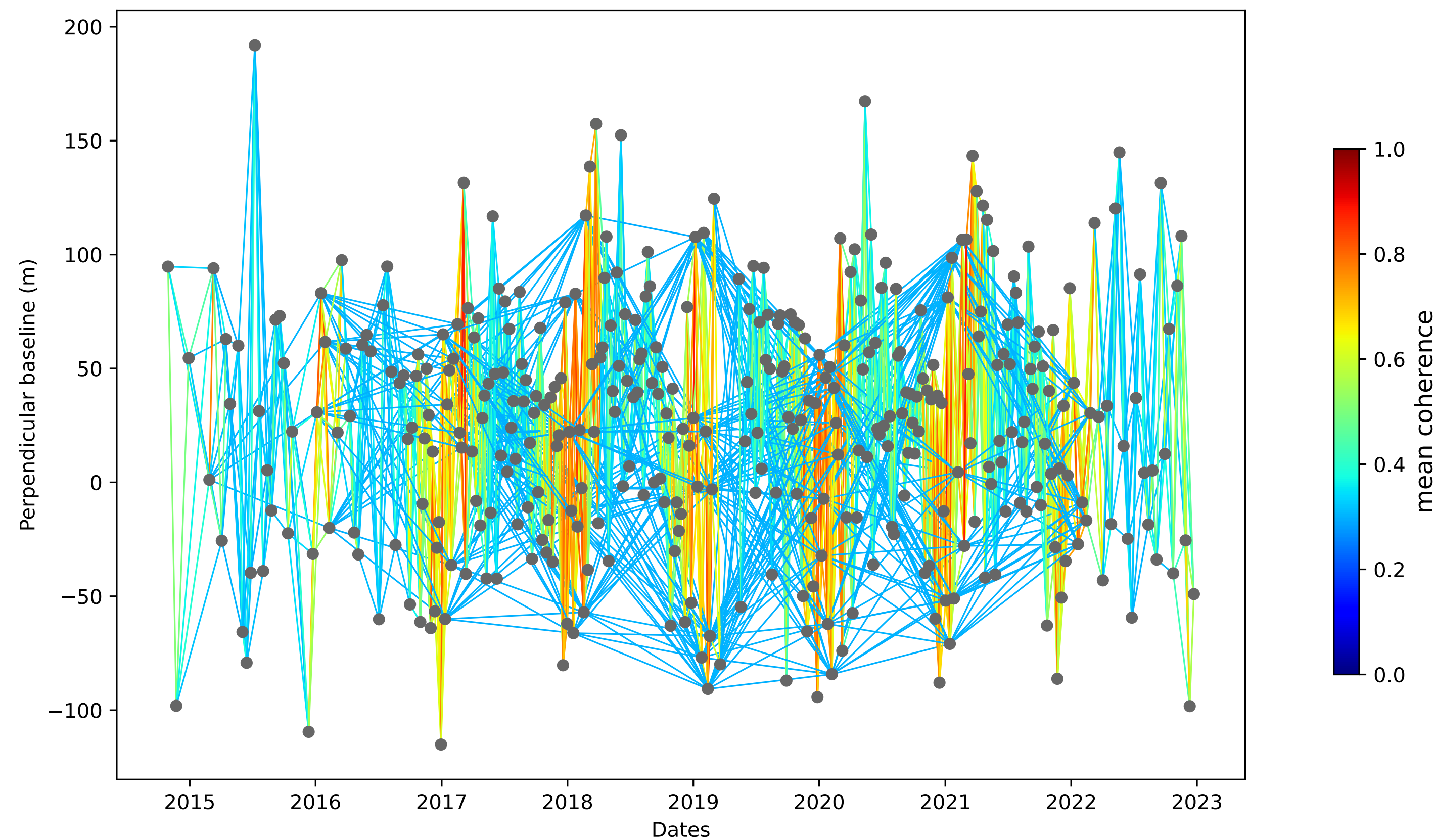
- To correctly estimate phase change (incl. deformation), we need an error model for ϵ_{ij}



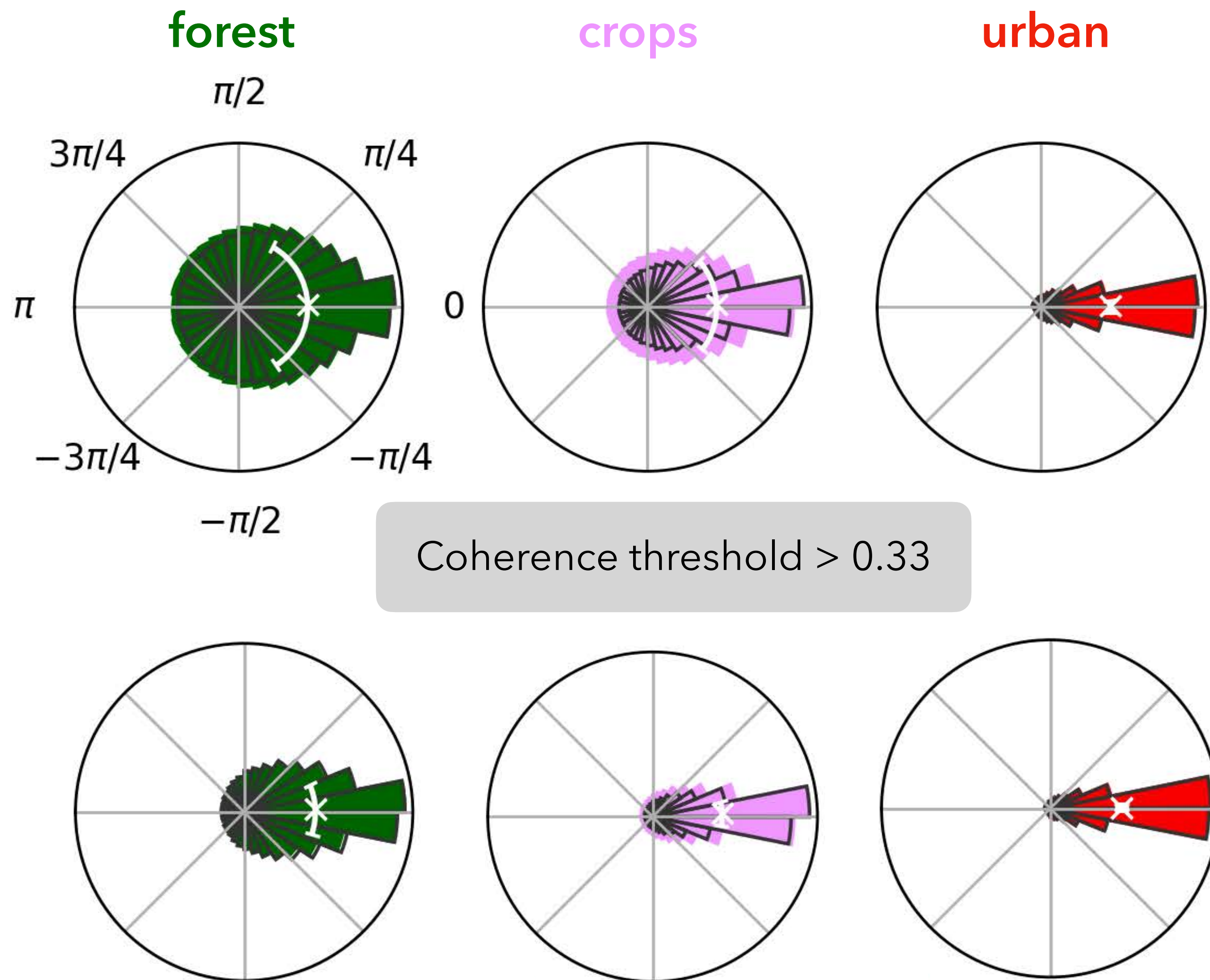


Data

- Sentinel-1 single burst x3
- Ground types from ESA WorldCover (2021)
- SBAS network including long-baselines between most-coherent months.

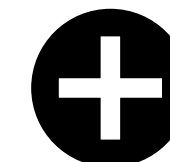


Spatio-temporal statistics of closure phase



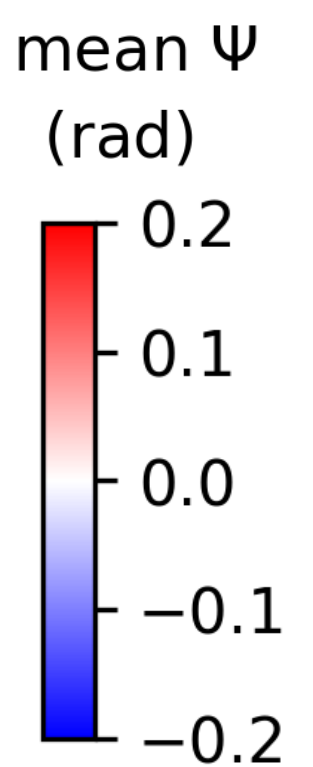
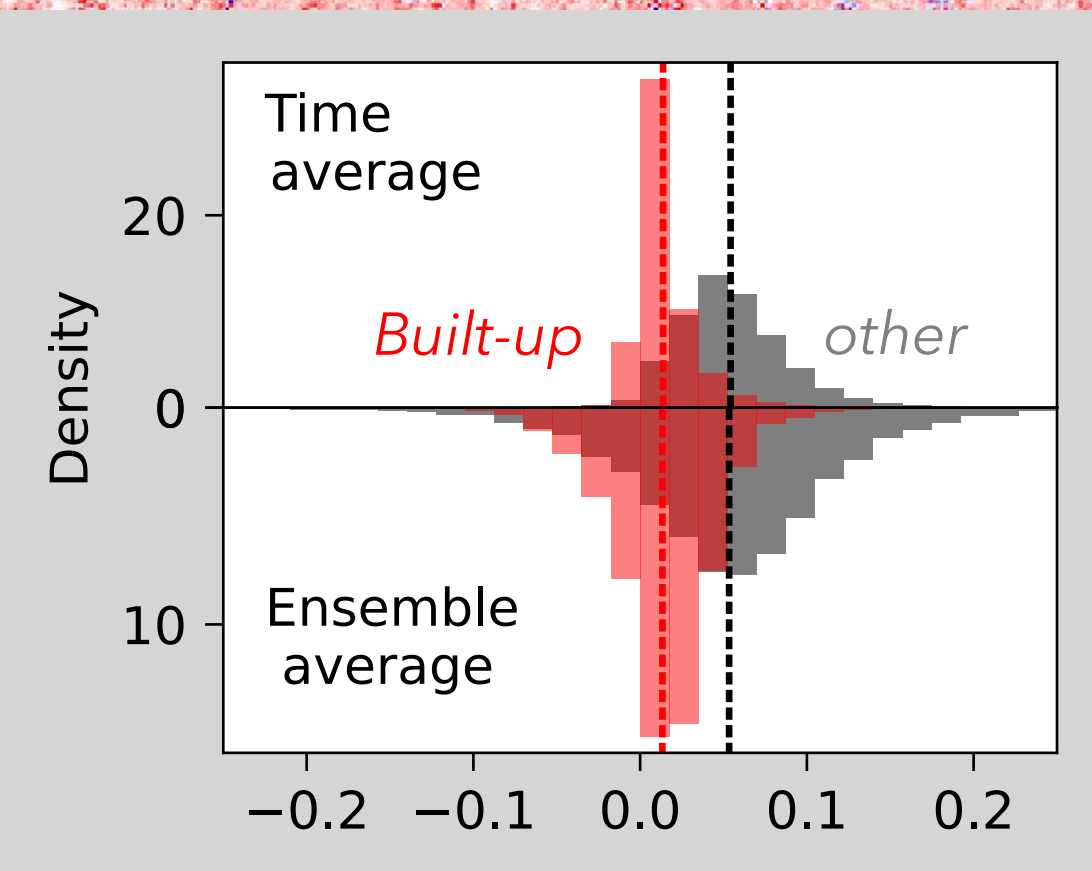
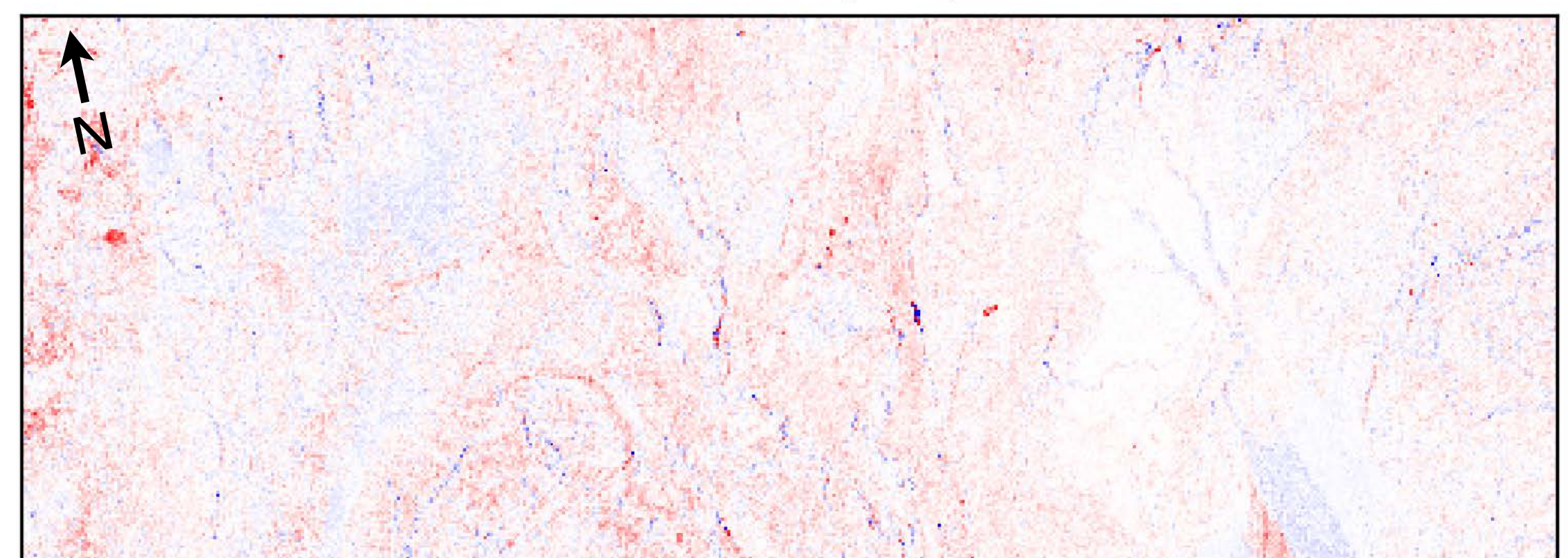
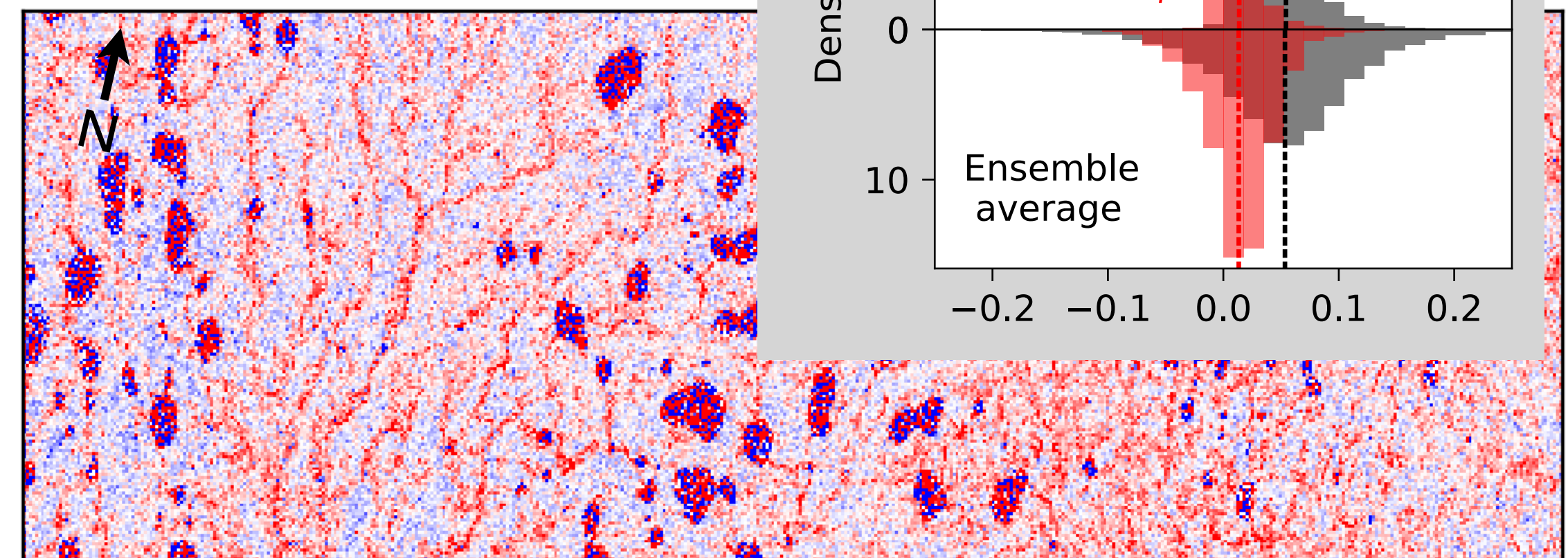
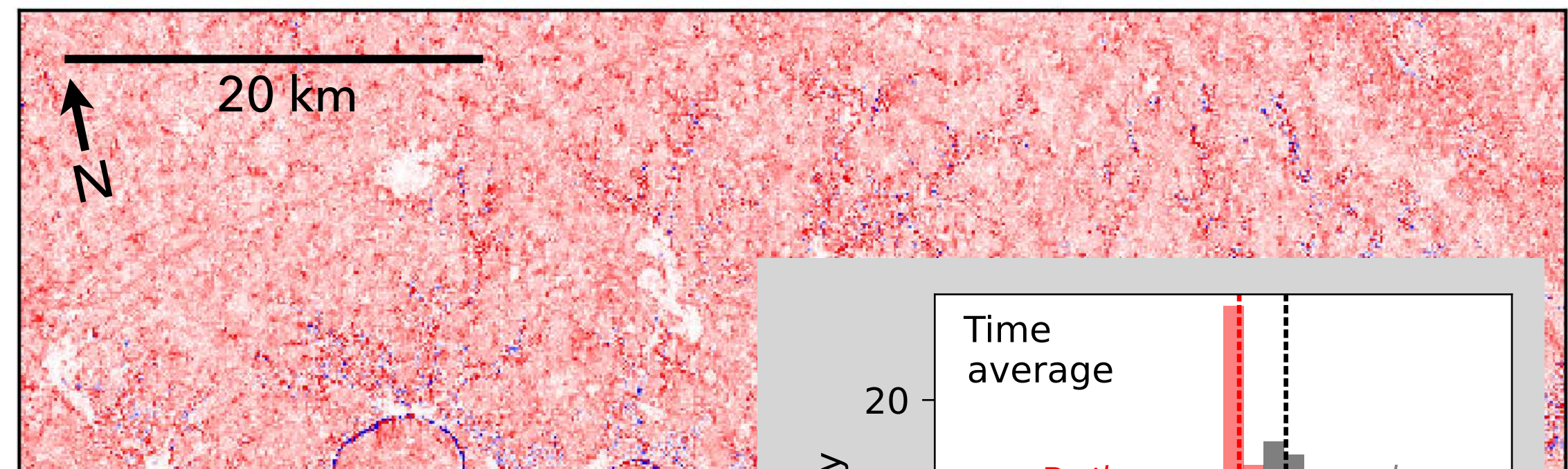
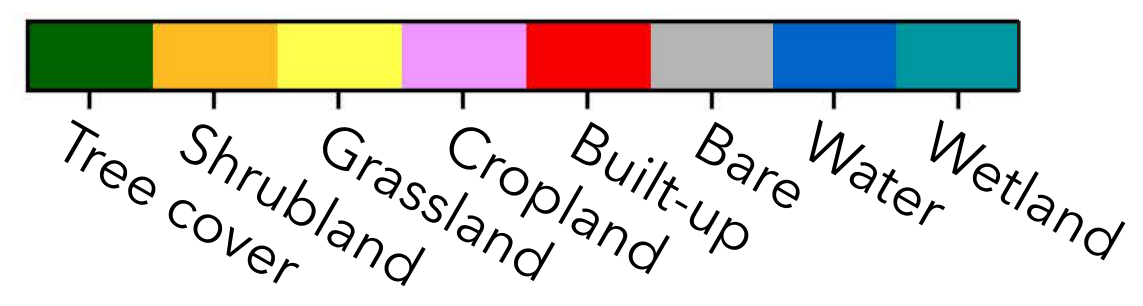
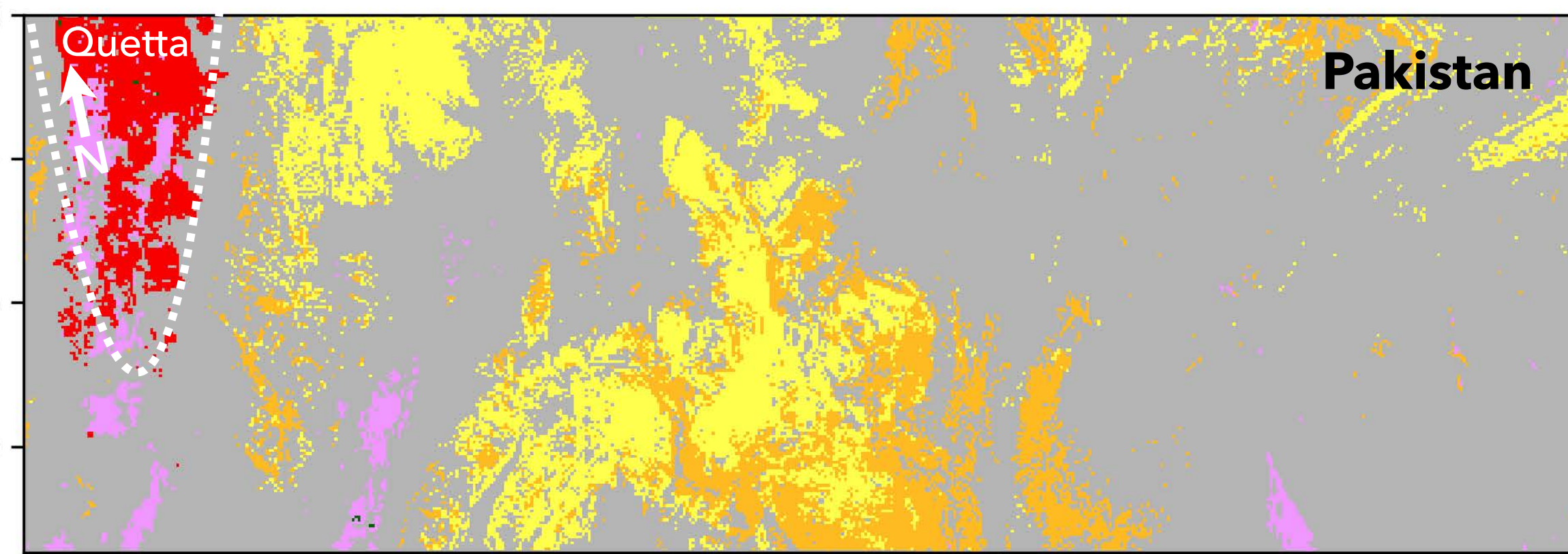
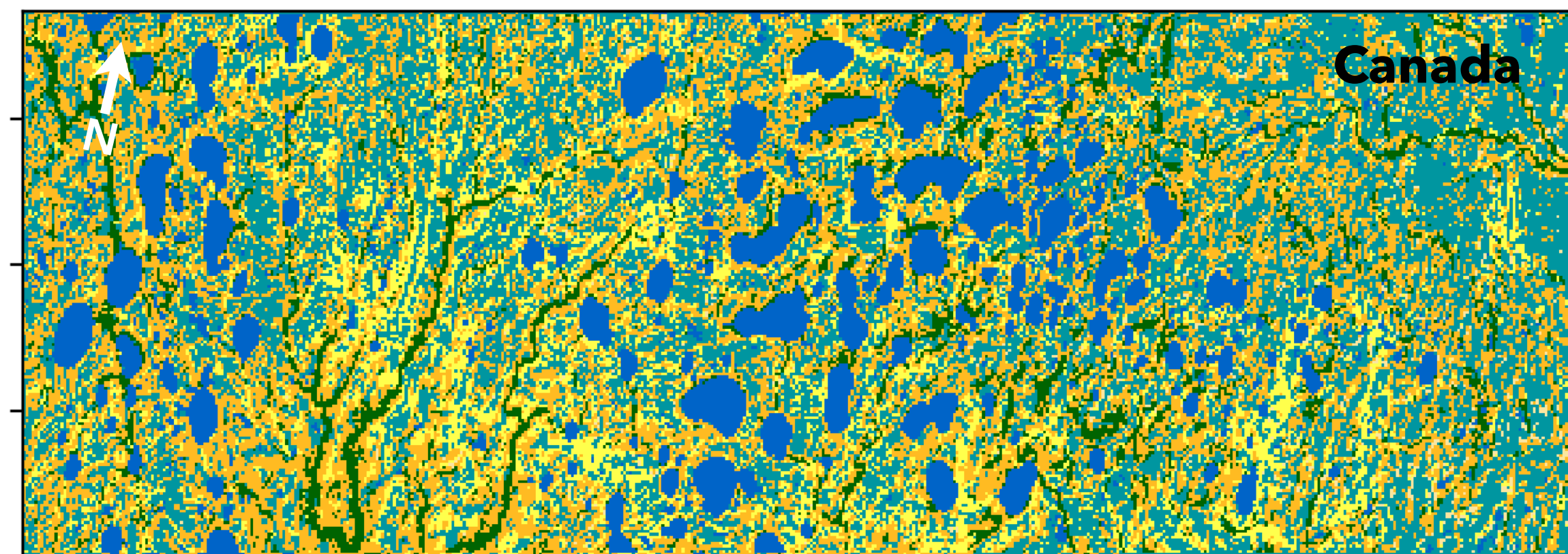
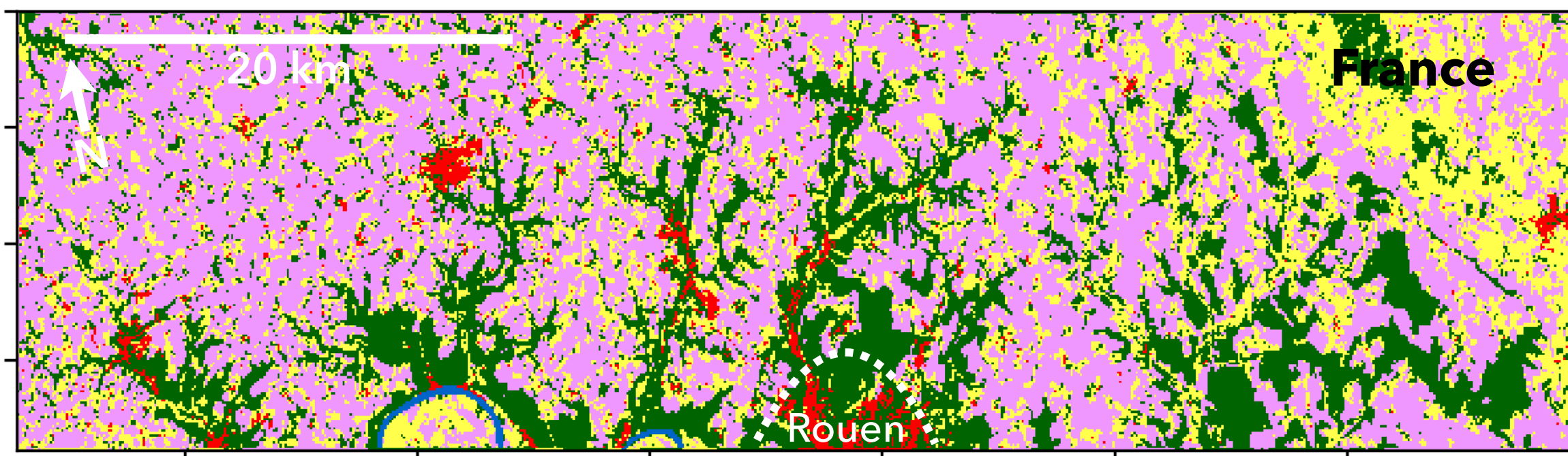
Bias in
« deformation »

Systematic error
> nonzero Mean

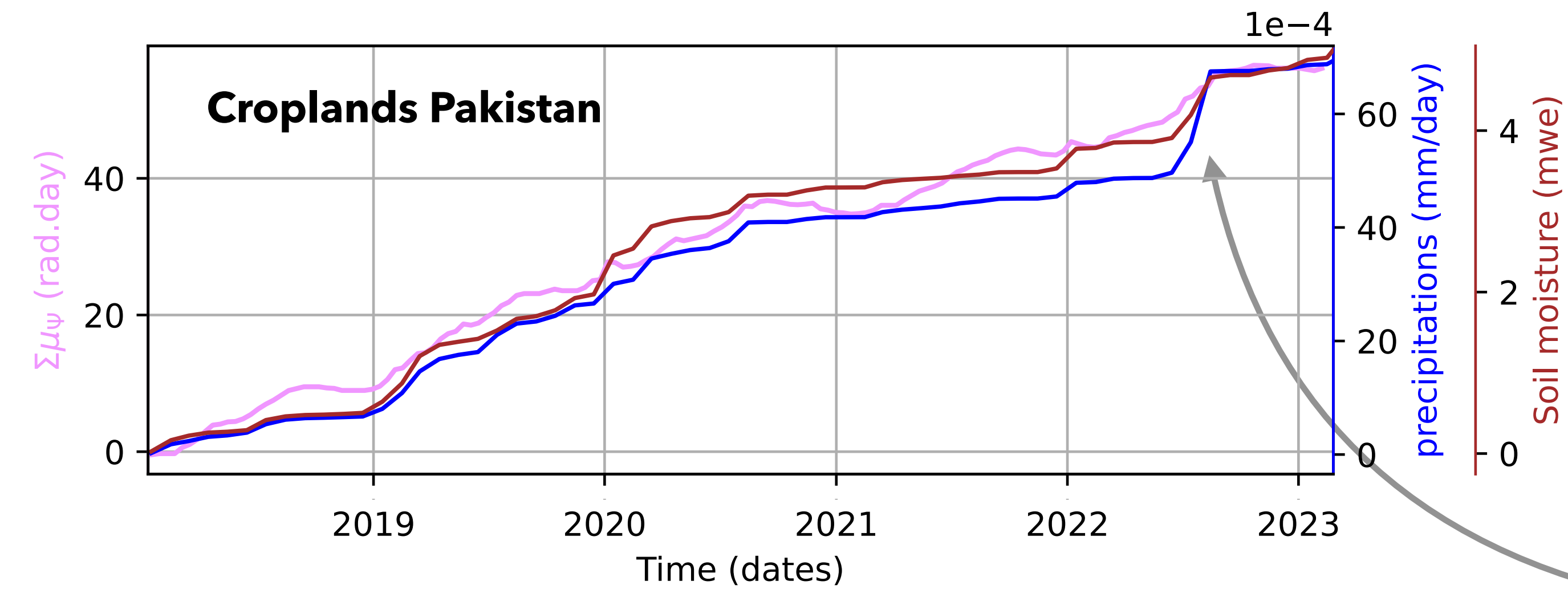


Noise
> Variance

- Evaluate the effect of land covers, number of looks, coherence, seasons...



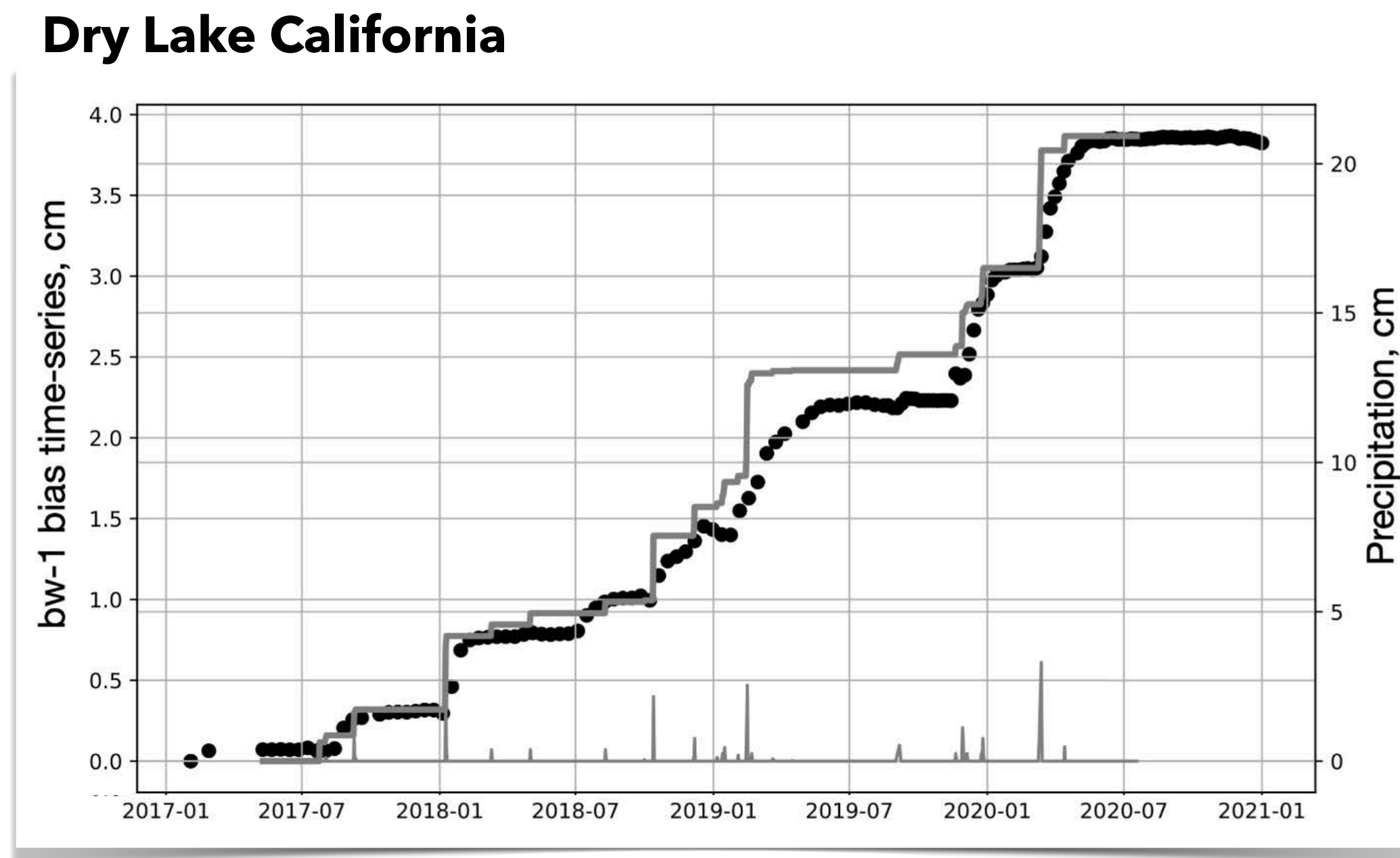
Maps of pixel-wise mean closure phase



Error time series: rain in dry climate



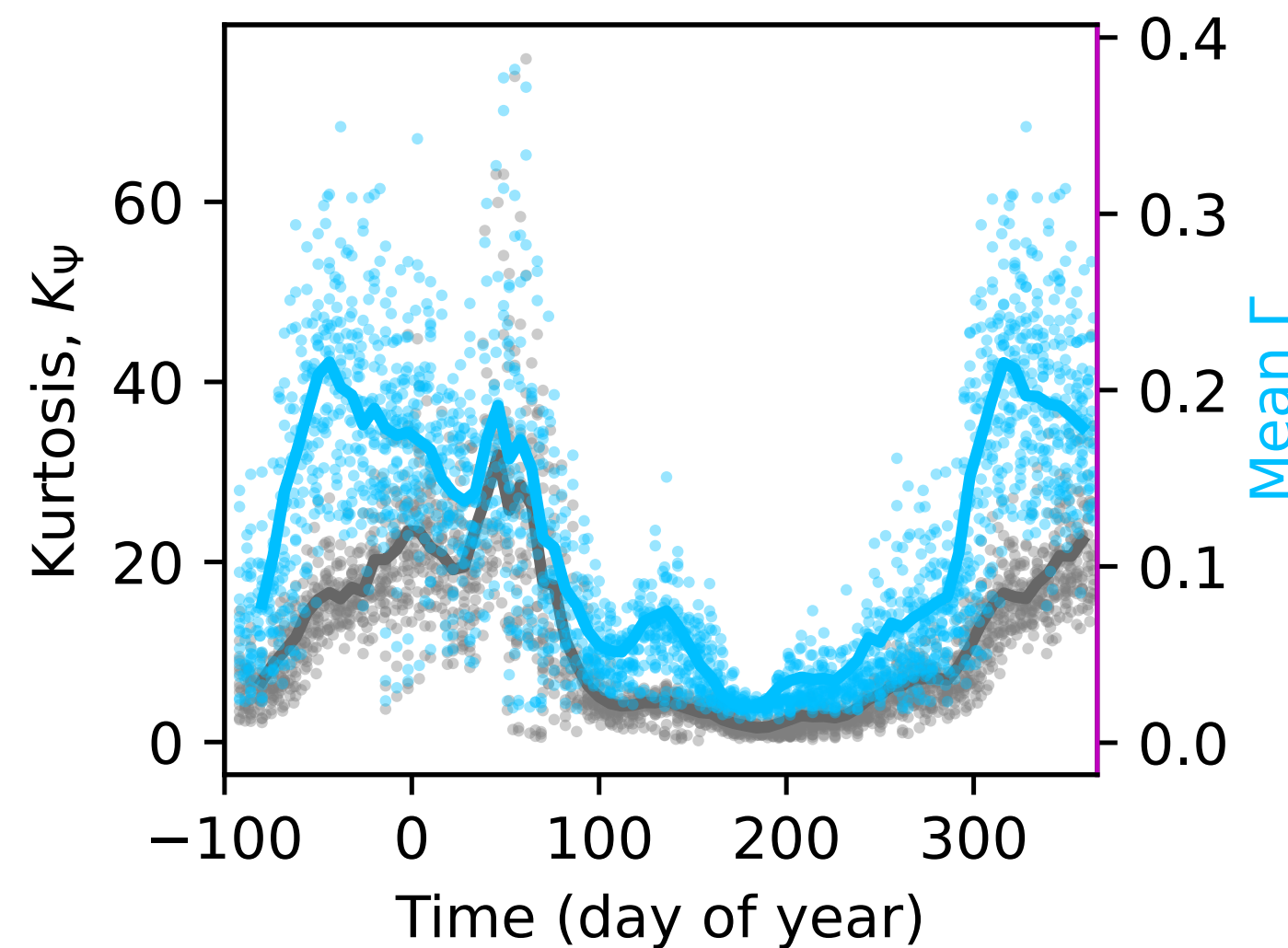
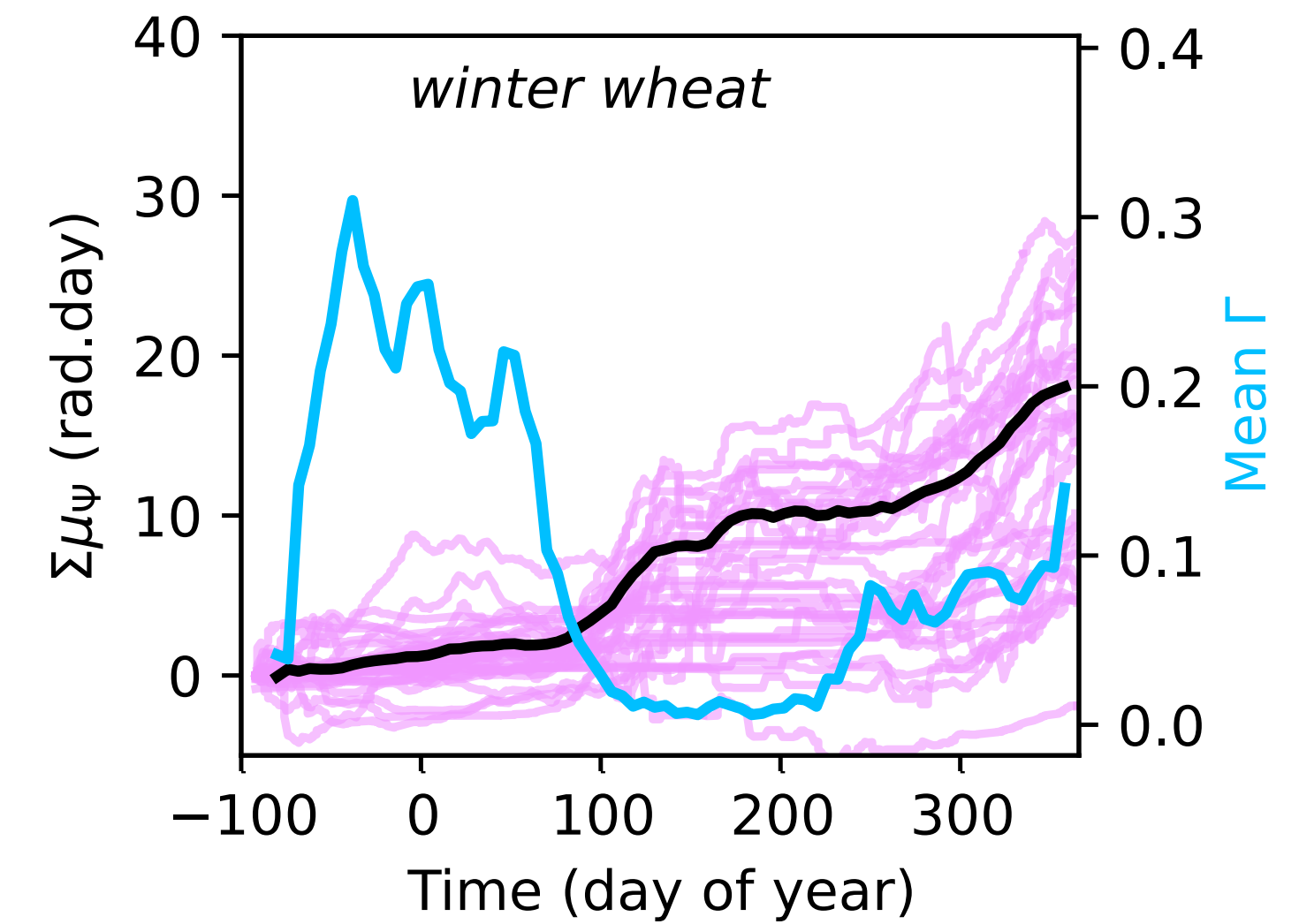
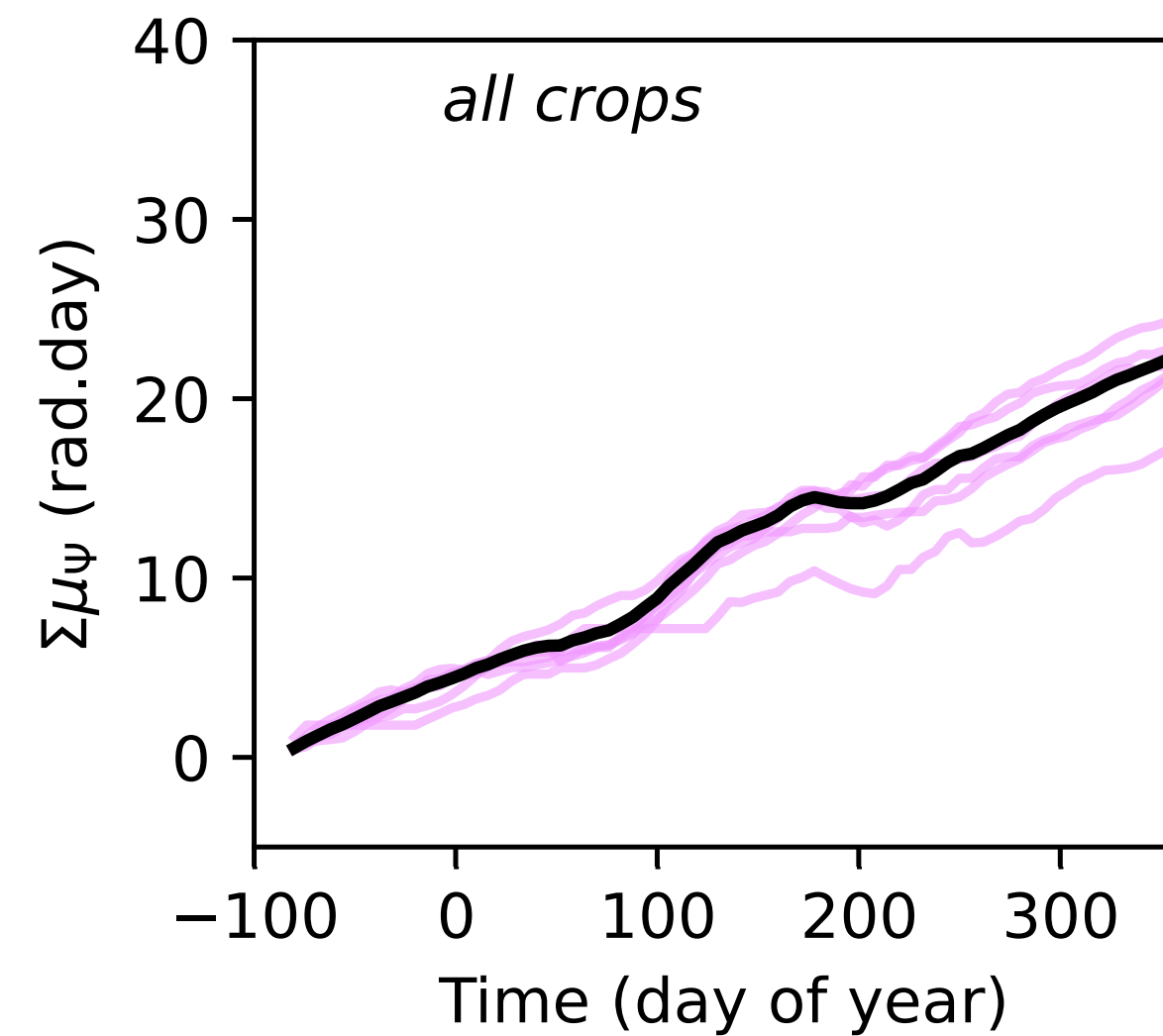
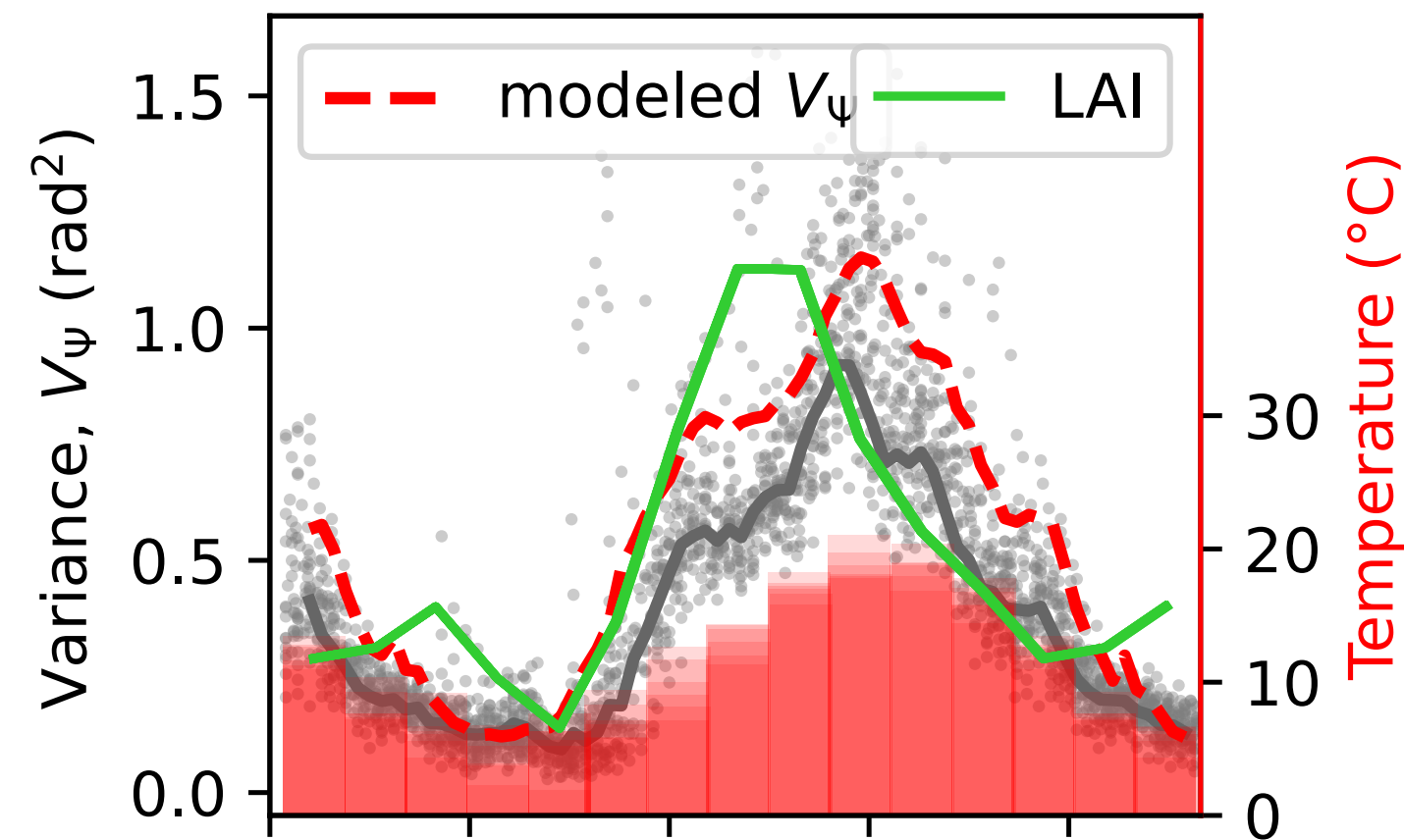
- The cumulative error co-variate with cumulative precipitation over **croplands in Pakistan**



Zheng et al. (2022)

Error time series: biomass growth in temperate climate

Croplands France



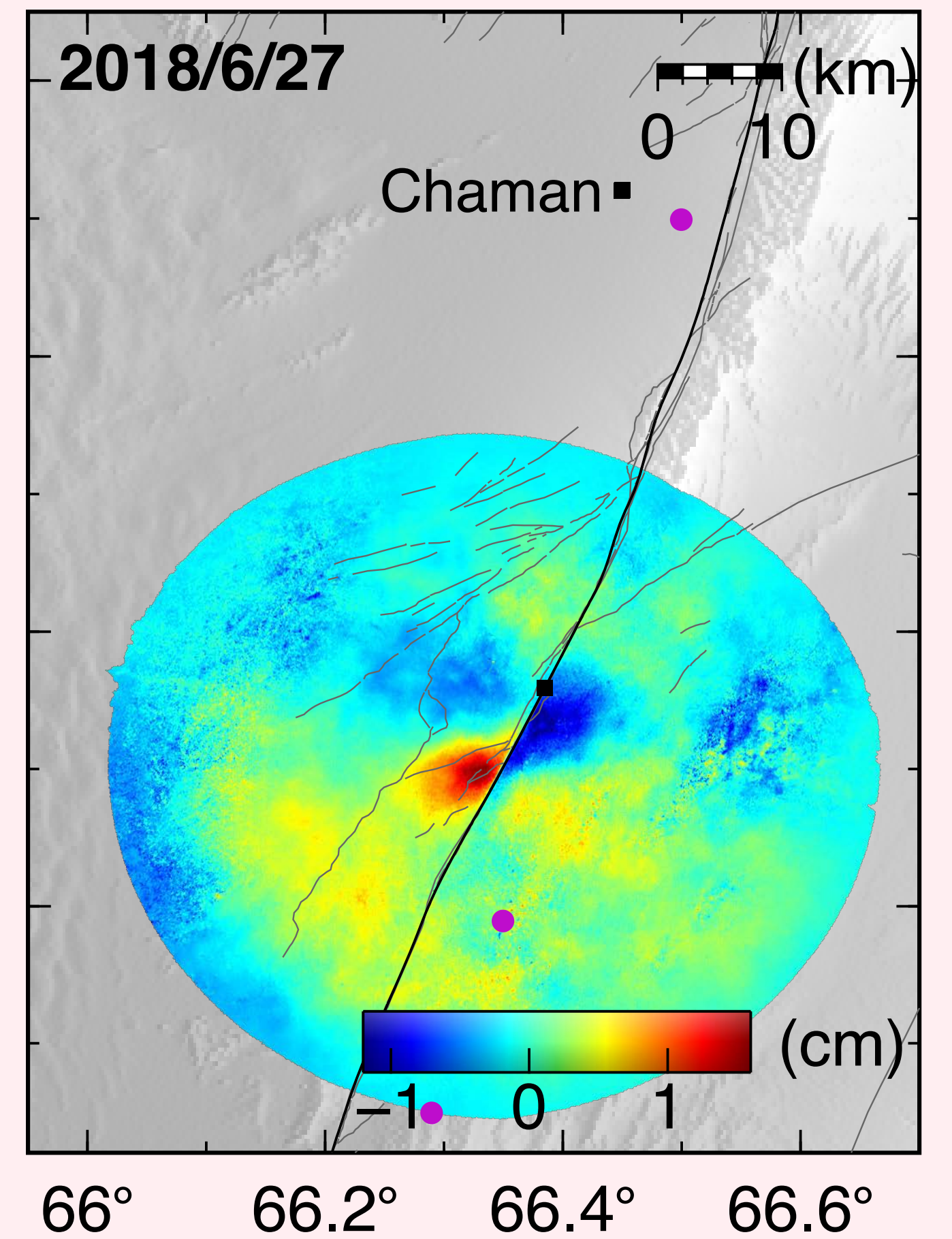
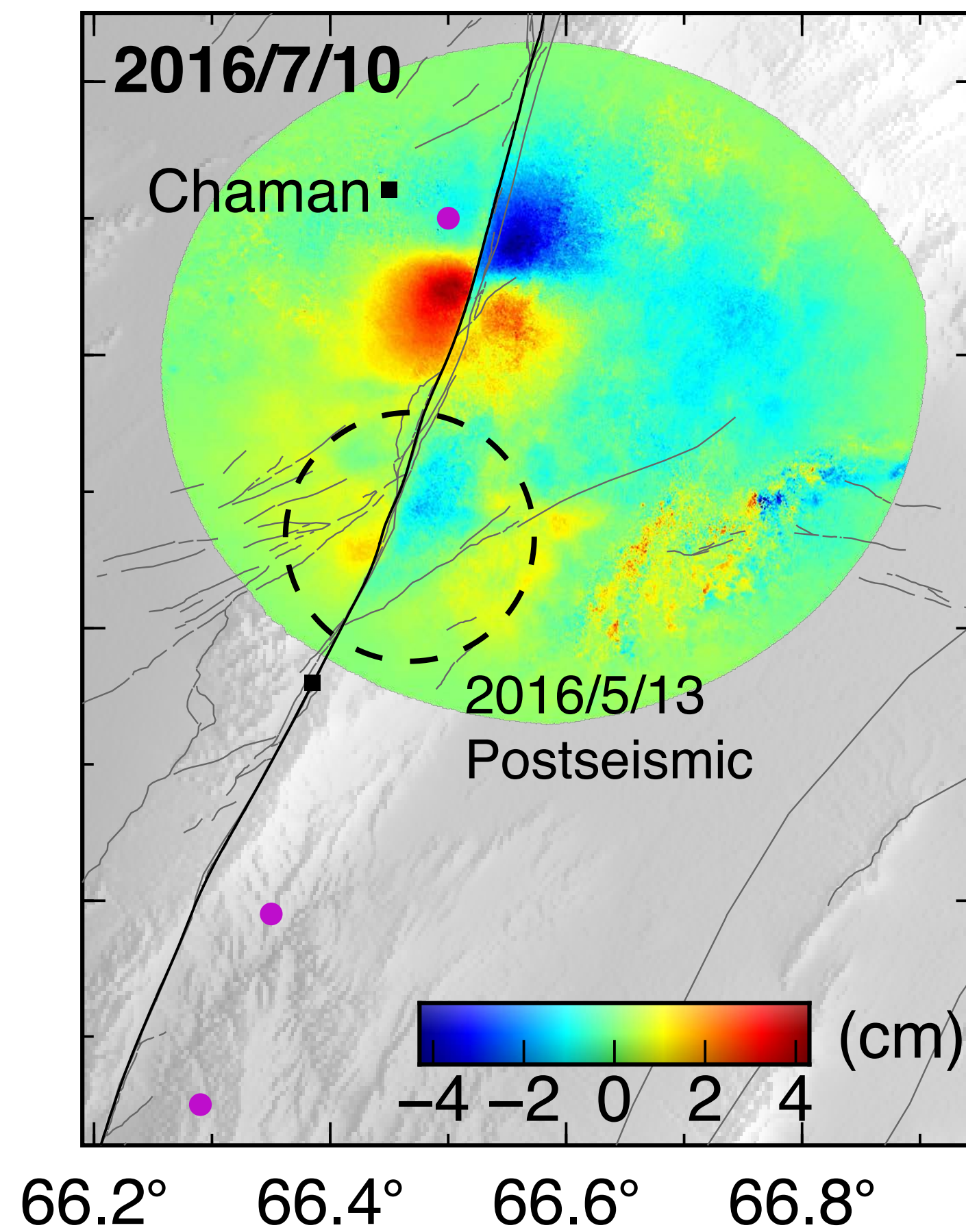
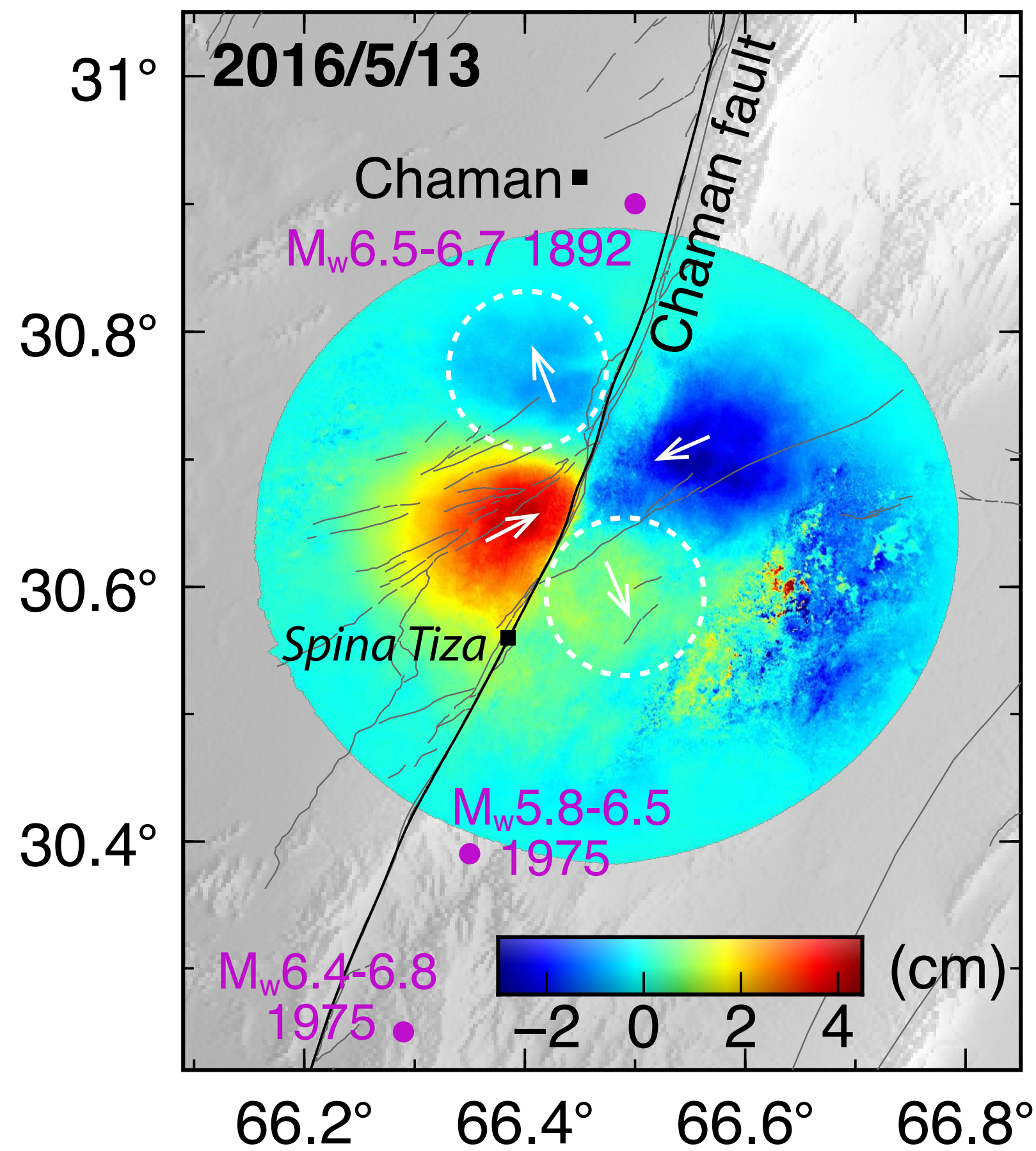
- Marked seasonality over croplands in France with field-specific bias accumulation depending on the type of crops

Backscattering model
for plant growth



In this presentation

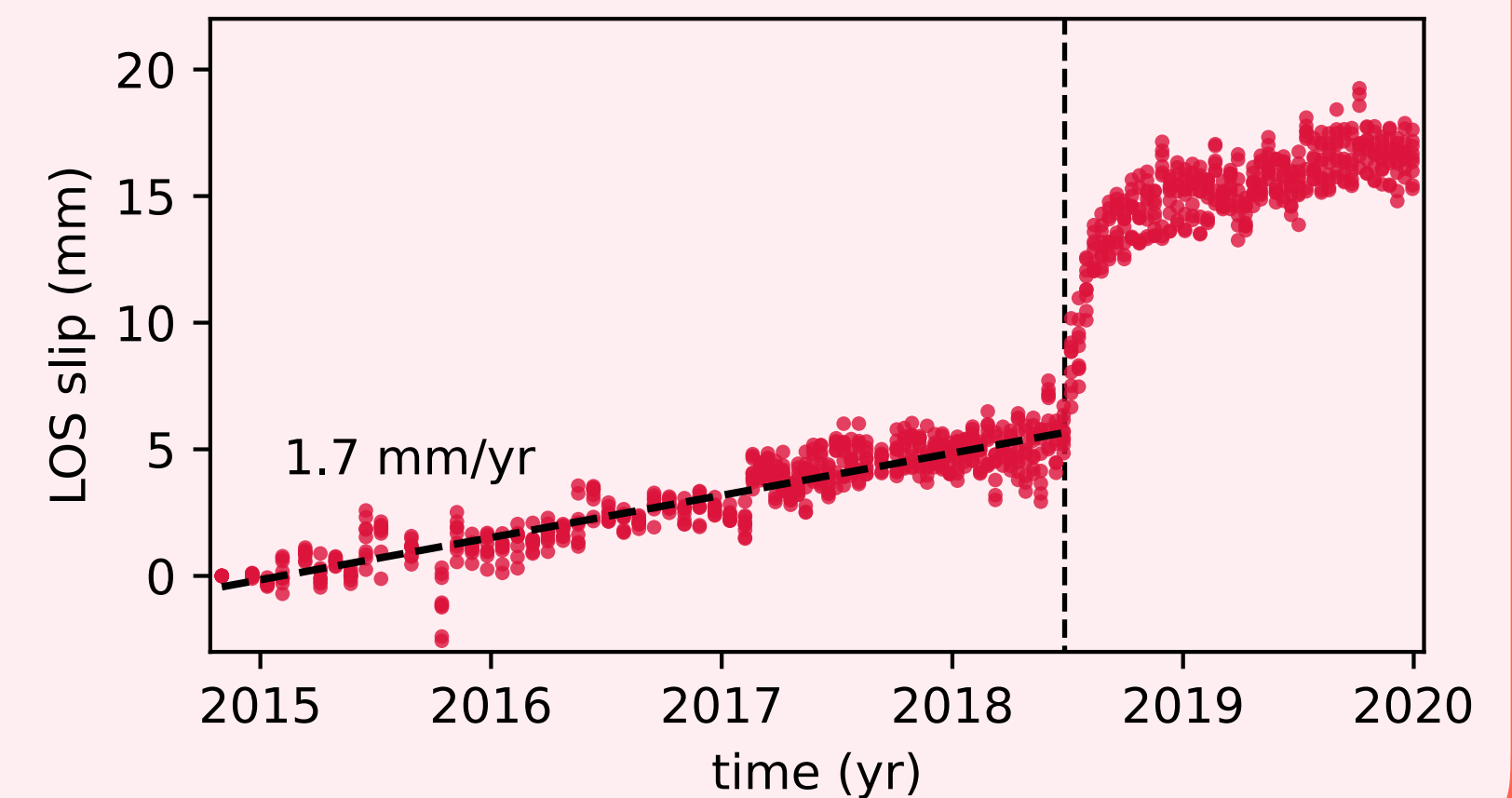
- A. Introduction to the design and parameterization of the Kalman Filter Time series analysis for InSAR
- B. Discussion on the complex nature of errors in InSAR phase change
- C. Some examples of deformation monitoring**
- D. Limits and conclusions



Seismic M_w is 4.4 ± 0.2

InSAR M_w is 5.0

Earthquake sequences



Time dependent inversion of geodetic data

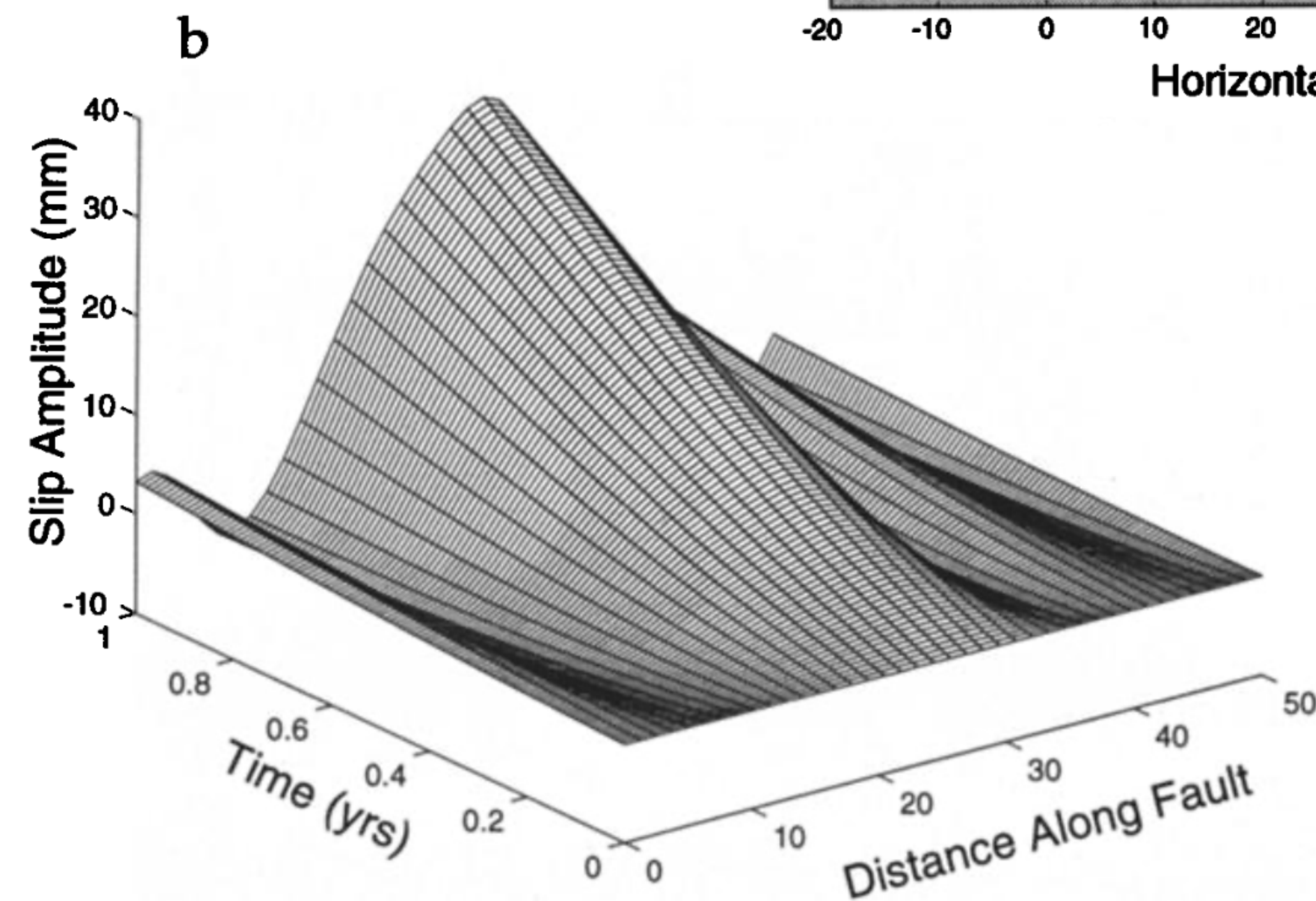
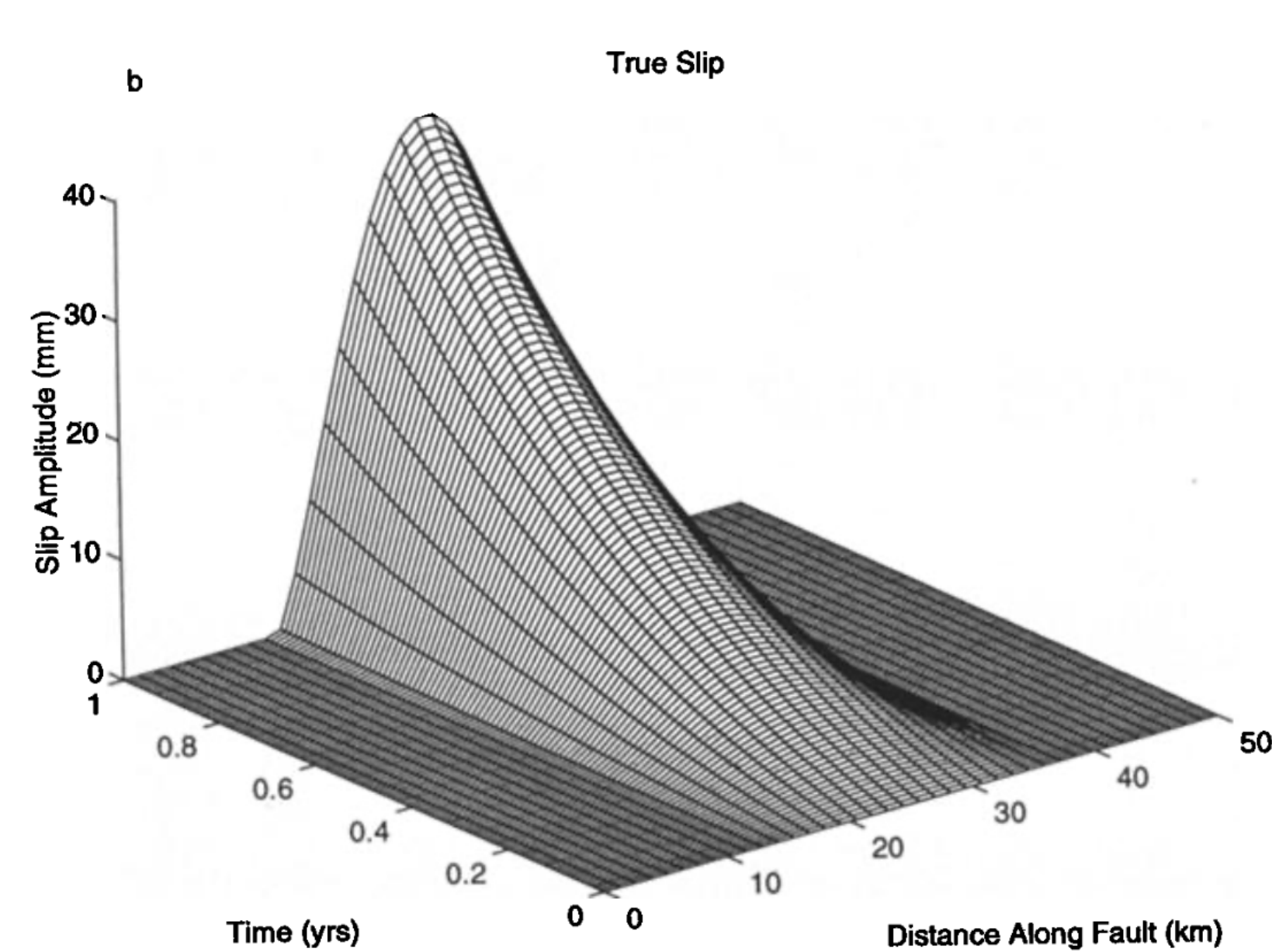
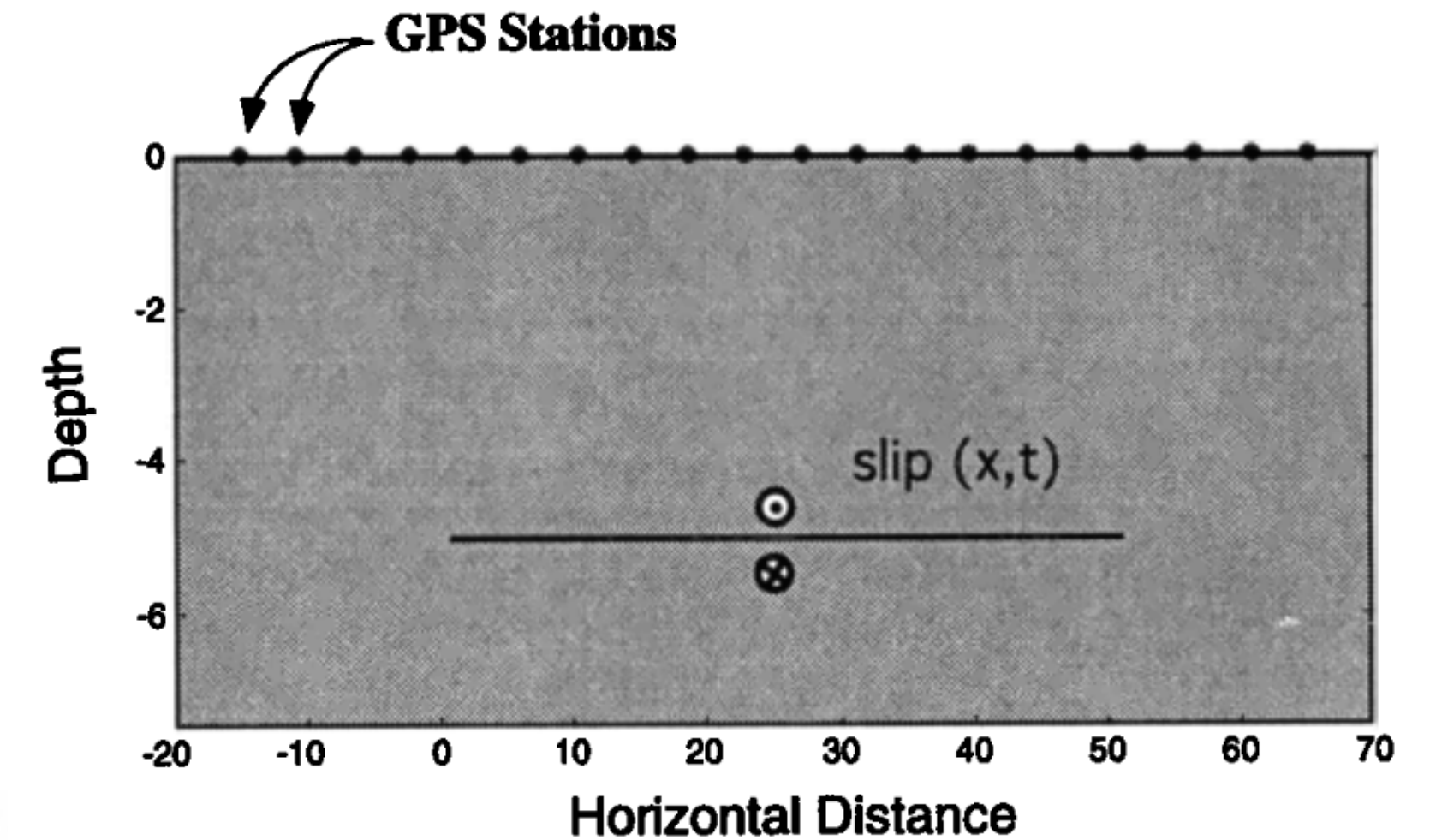
Paul Segall

Department of Geophysics, Stanford University, Stanford, California

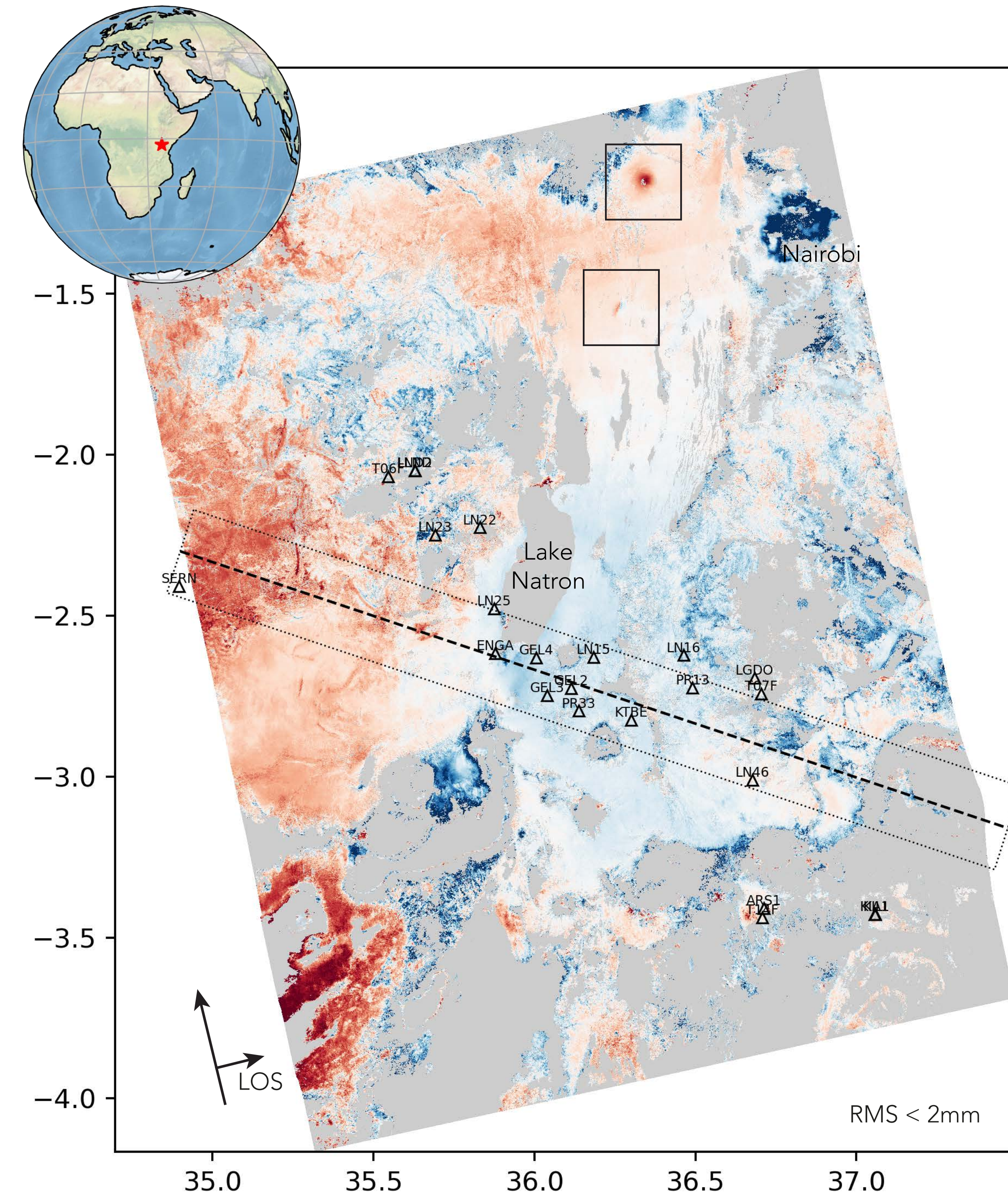
Mark Matthews

Department of Mathematics, Massachusetts Institute of Technology, Cambridge

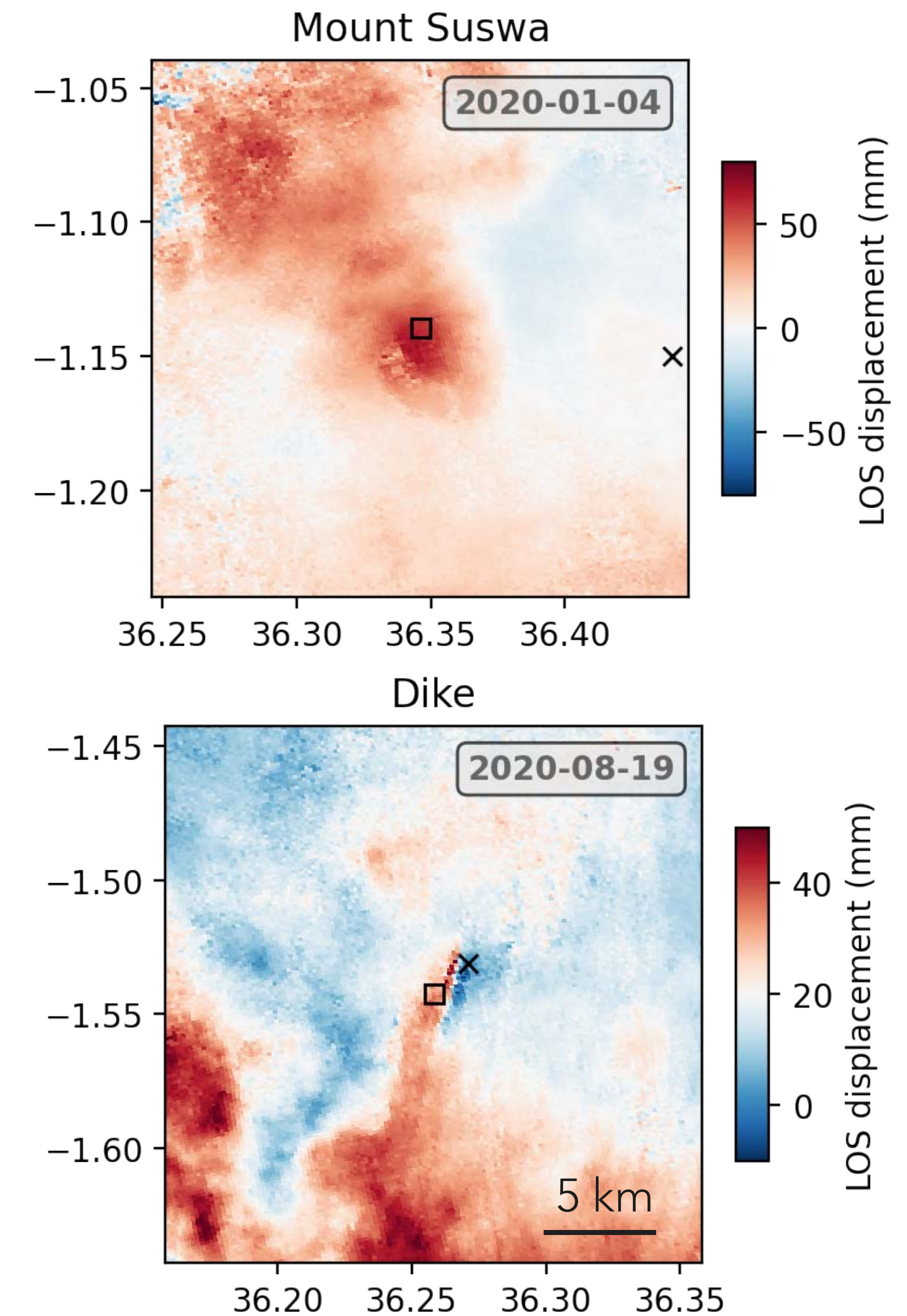
1997



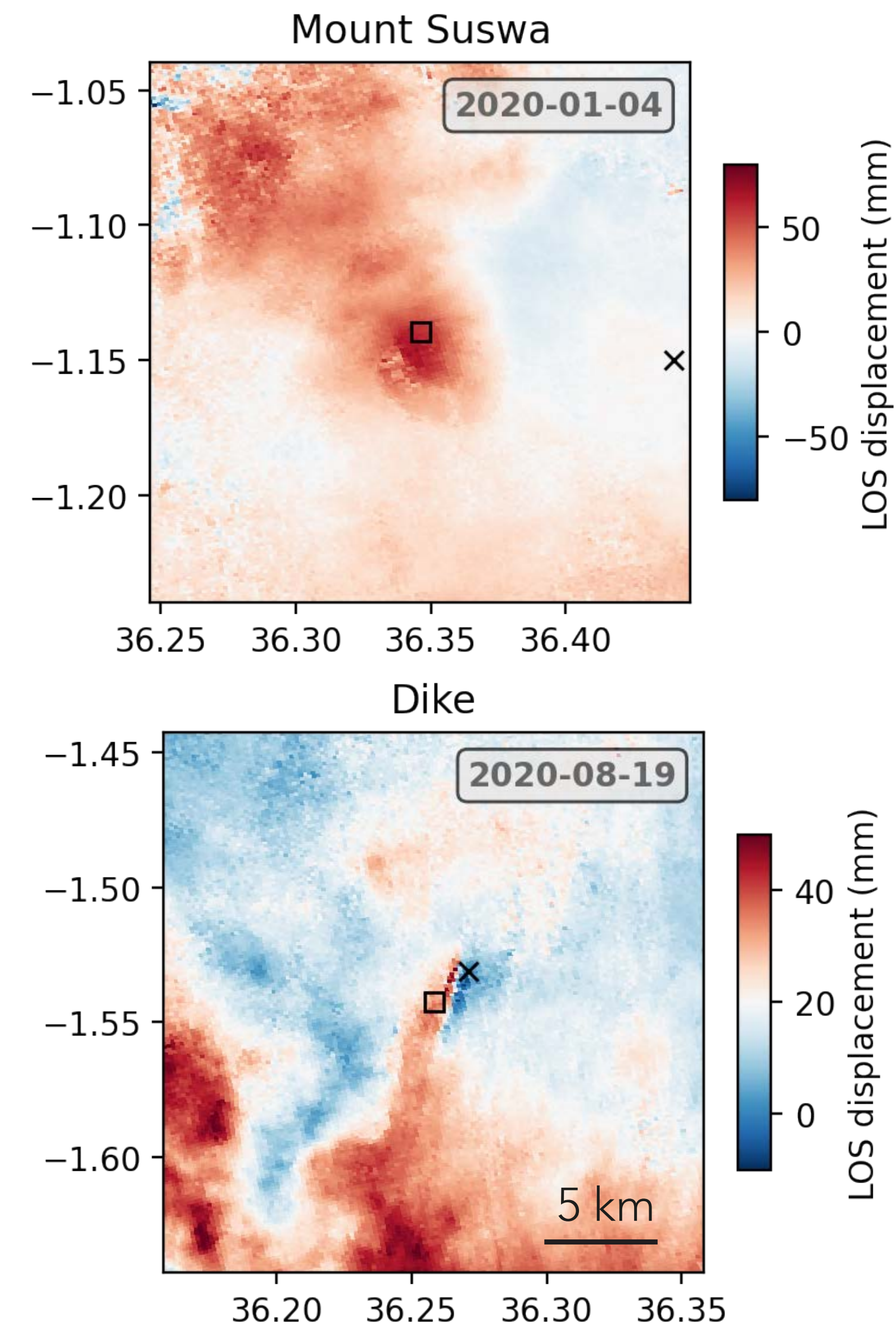
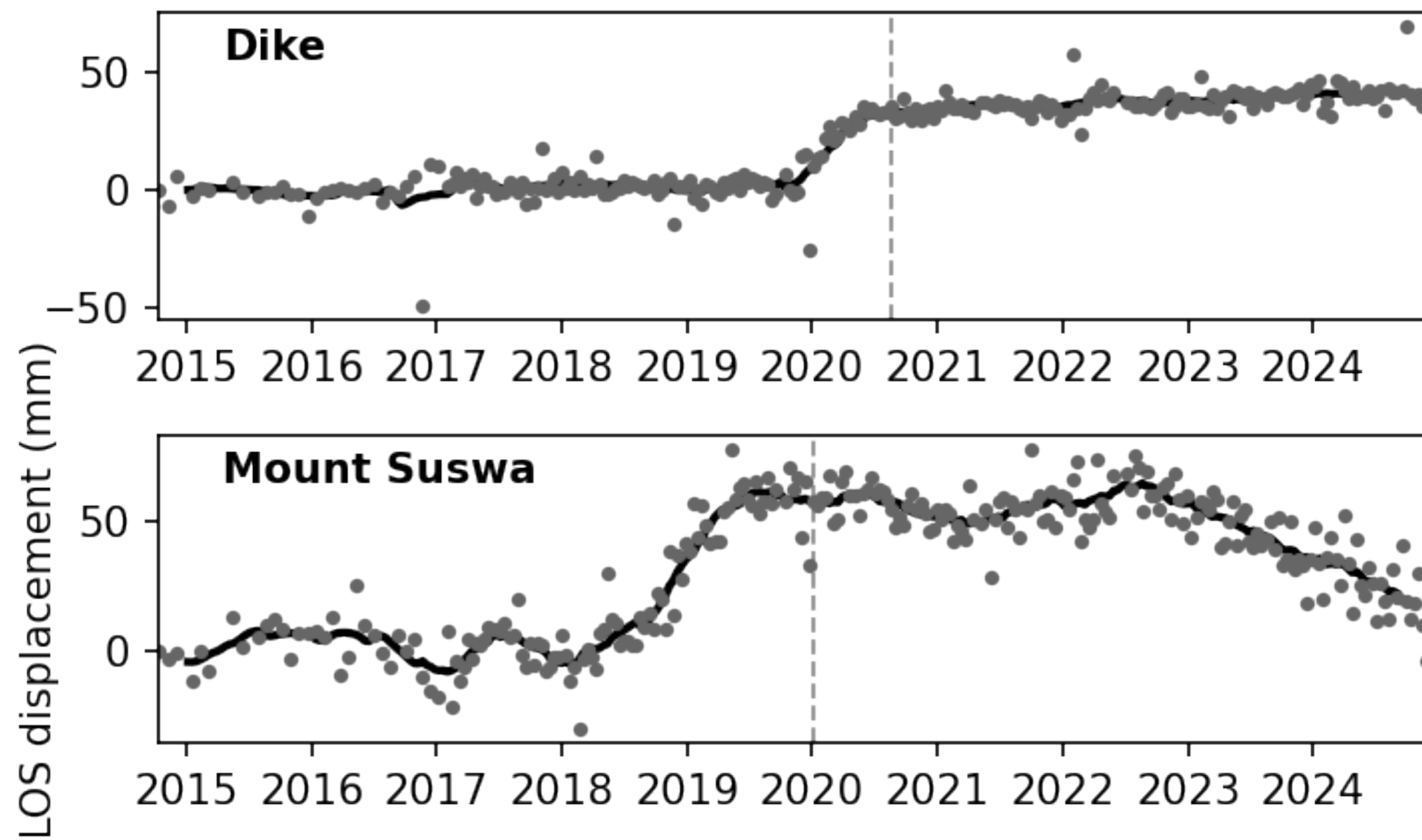
- Network Inversion Filter (Kalman Filter here) is capable of imaging fault slip transients, including propagating slip events



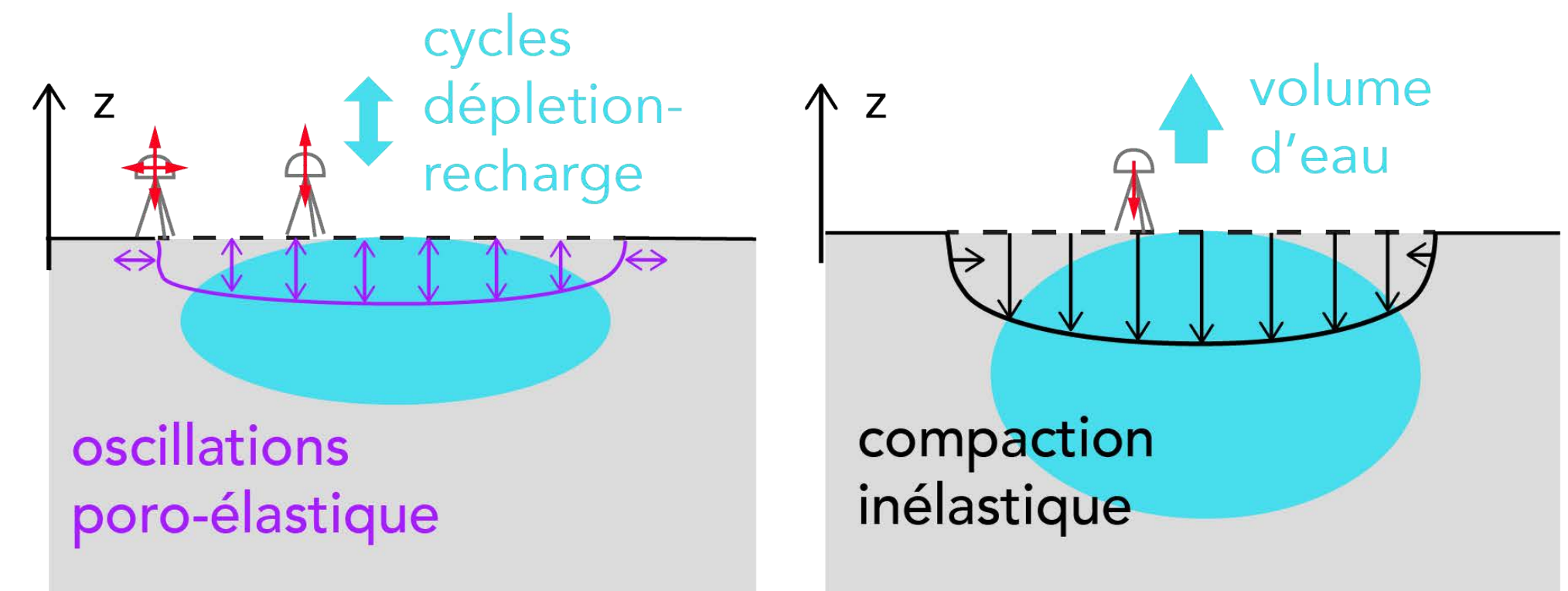
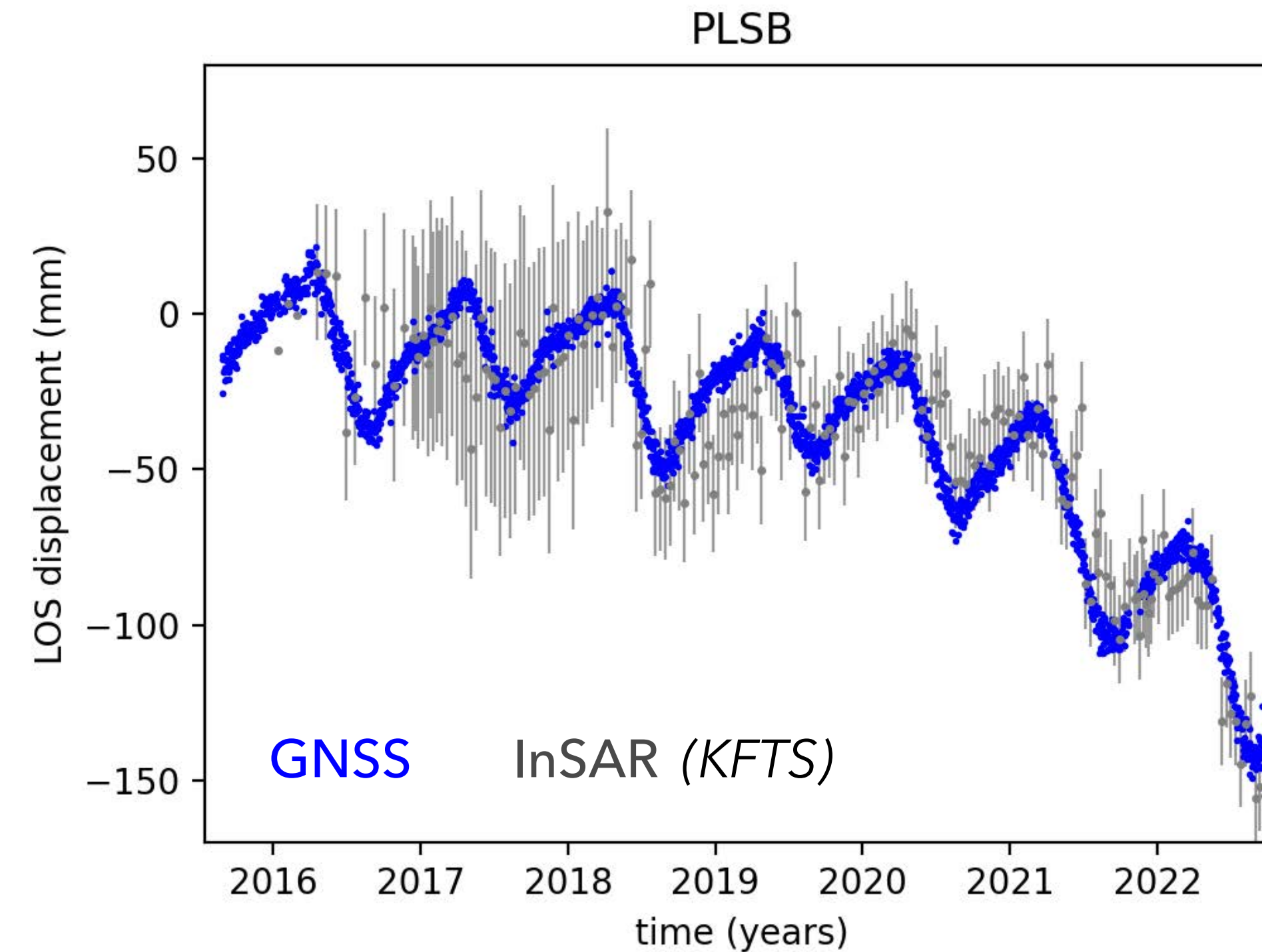
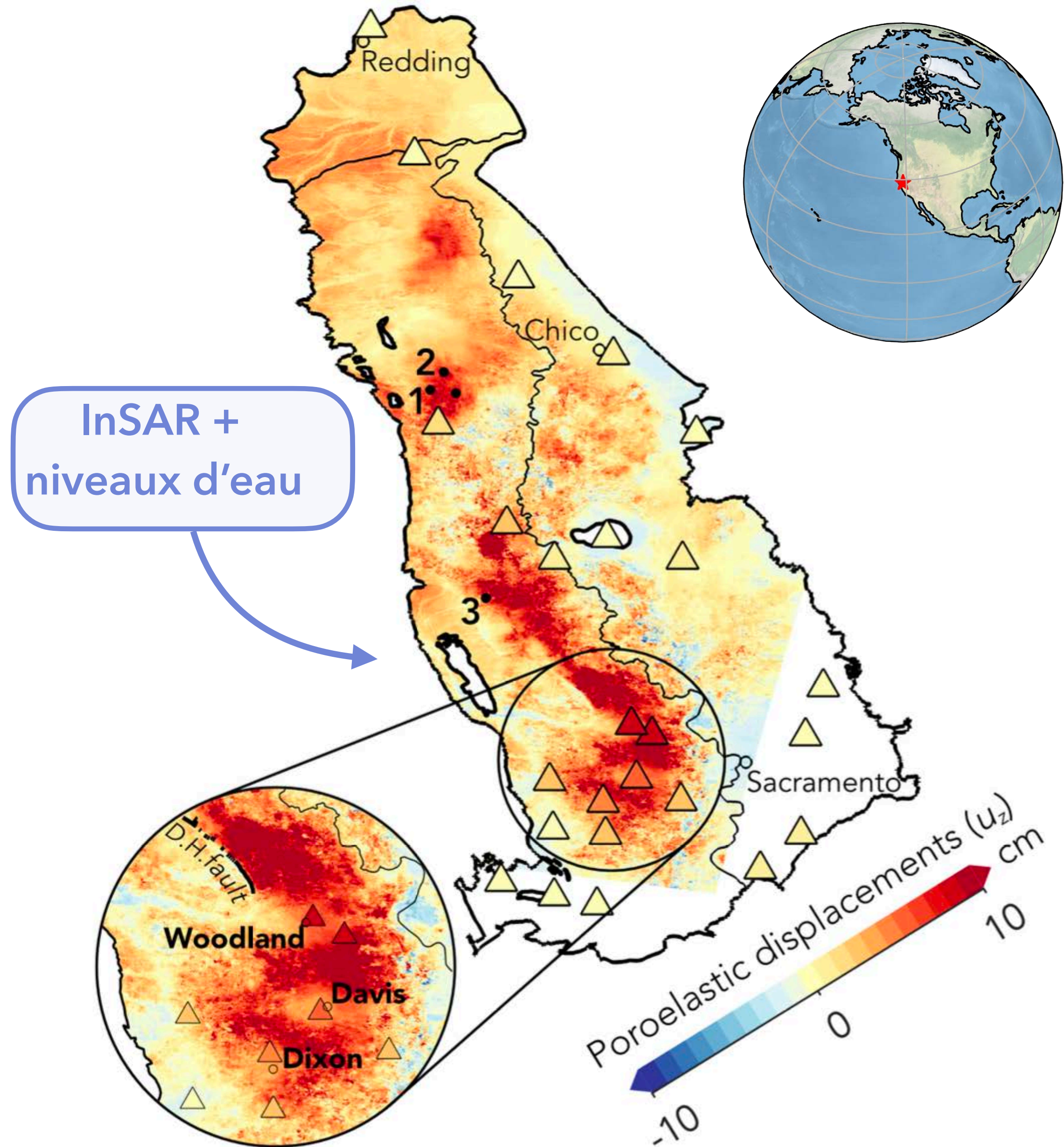
Magmatic activity



Magmatic activity

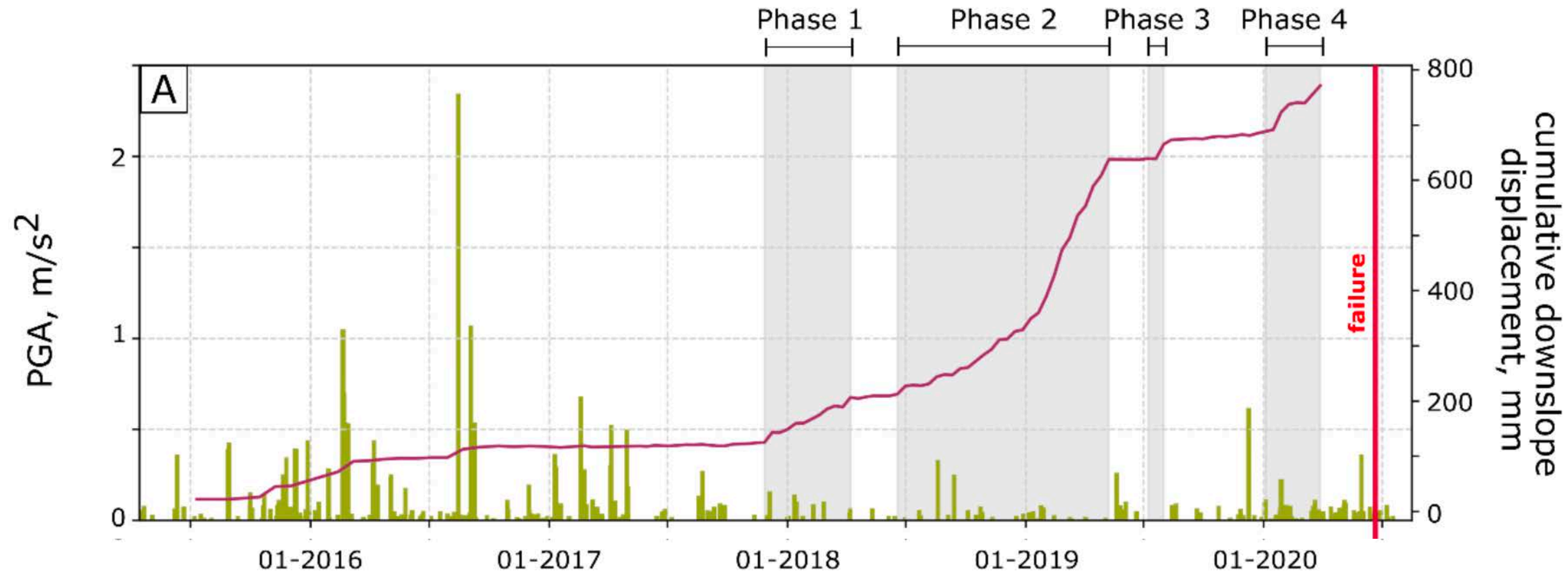


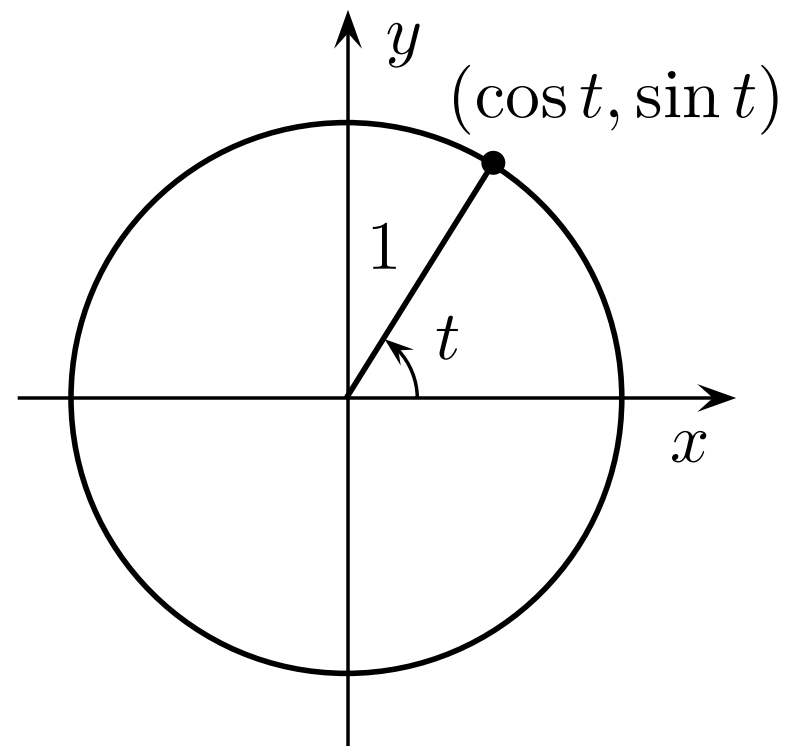
Poroelastic and inelastic compaction from groundwater pumping



Natural hazard perspective

- Anticipating landslide failure ?





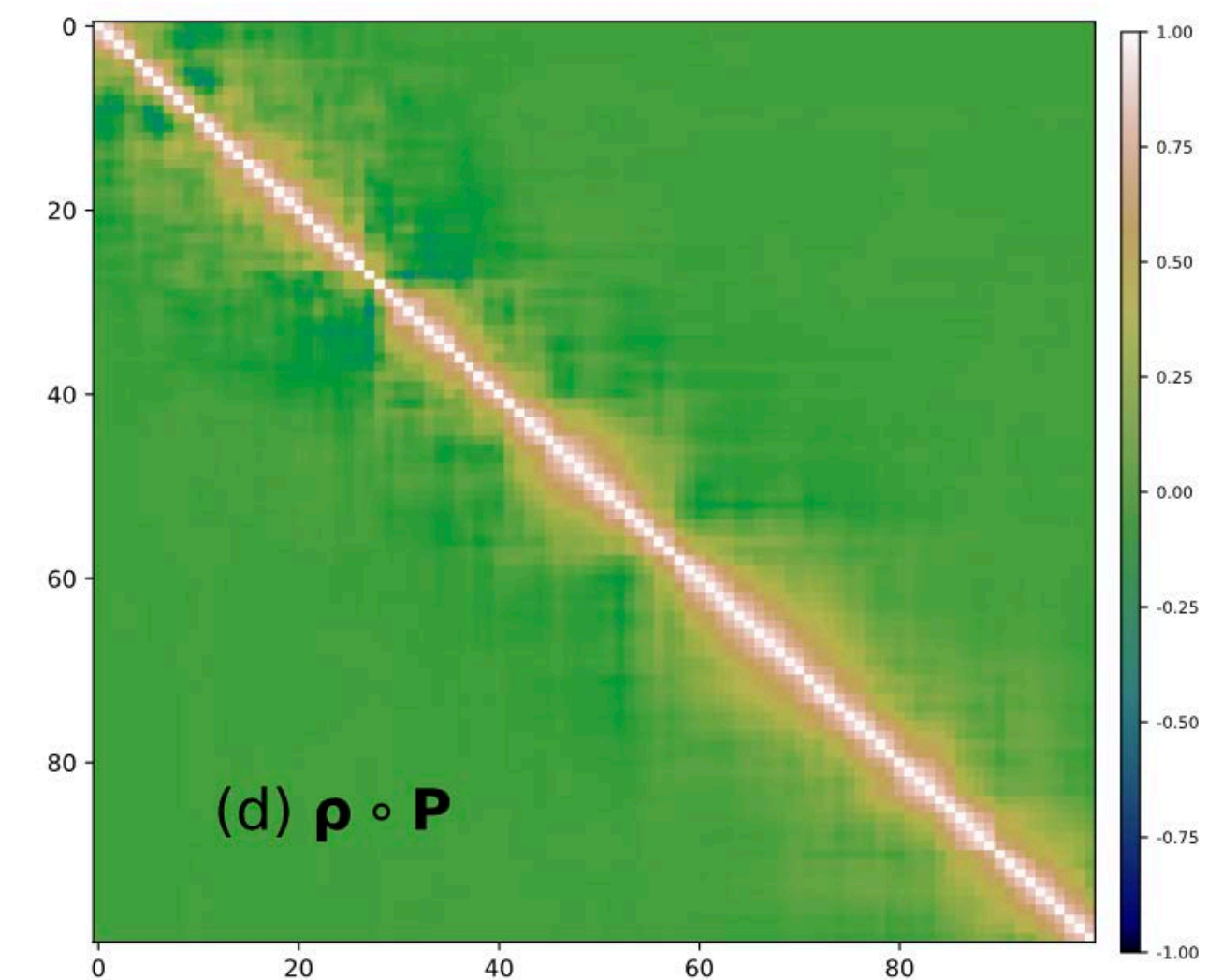
Potential for improvement

« *Network Inversion Filter leads naturally to automated methods for detecting anomalous departures from steady deformation* »

Segall & Matthews (1997)

- Improve the forecast model on the fly
 - Evolving parametric description that can be constantly assessed tracking the gain and innovation.

In practice : hard to calibrate and still requires human assessment.
- Upgrade to smoother for backward propagation of model parameters
- Add instantaneous velocity to the state
- Work with wrapped InSAR phase ? 2π ambiguity resolution using integer least squares (ILSs)
- Optimize covariance localisation



Carrassi et al. (2017)

Wang, Brouwer et al. (2026)

Conclusions

- Although simple, Kalman filtering, can produce operational and rapid updates of preexisting time-series of deformation from ongoing satellite observations and linear descriptive models
- The sequential update is suited for continuous assessment of the model and variance-covariance matrix representativity. Gain and innovation are quantity of interest
- InSAR deformation monitoring can get tricky over vegetated grounds, where a complex non-zero mean error model is required.
- Multiple interesting applications :-)

Shared services / S-CAPAD



Merci de votre
attention